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Chapter 21: String theory and particle physics

We will be using D-branes in a clever way:

• We need superstrings to get fermions in spacetime. (In fact a chiral set of fermions.) We will use the type IIA string (which is non-chiral!)

• Six compact dimensions to get from $D=10$ to $D=4$: these are T^6 with equal radii.

$$x^i \sim x^i + 2\pi R$$

• To get Yang–Mills in $D=4$ spacetime we need D-brane (stacks $\Rightarrow$ Yang–Mills).

Comment: In type IIA, IIB and M-theory there are no Yang–Mills fields before compactification, but in the other cases there is!

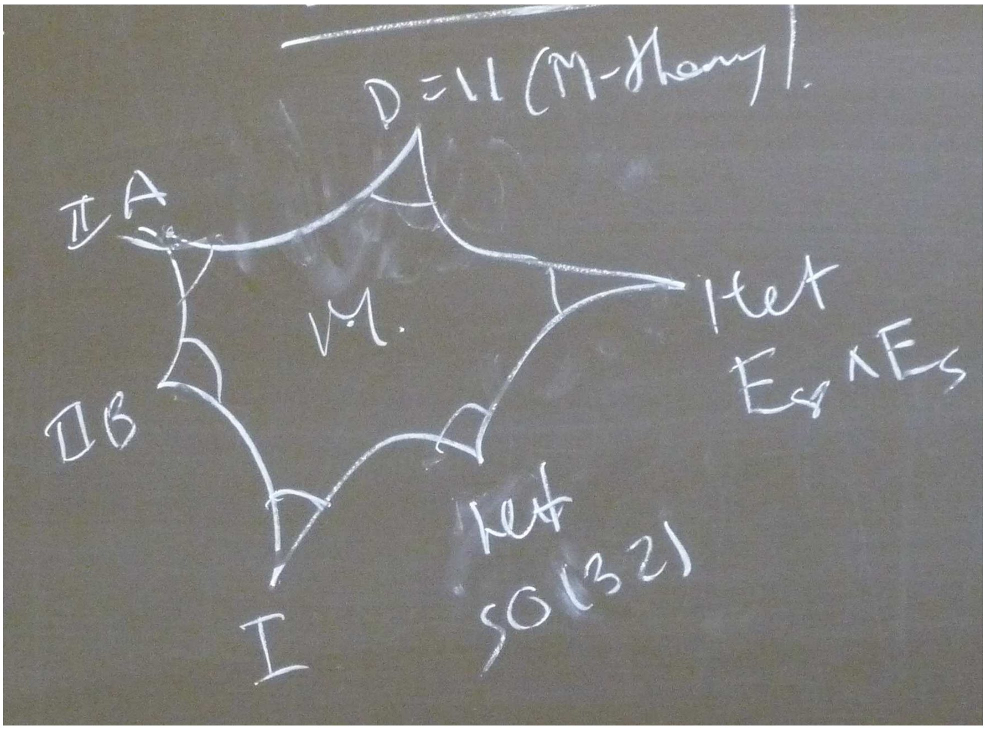

Figure 1.

$$A = \begin{pmatrix} \boldsymbol{e}_1 \cdot \boldsymbol{e}_1 & \boldsymbol{e}_1 \cdot \boldsymbol{e}_2 \\ \boldsymbol{e}_2 \cdot \boldsymbol{e}_1 & \boldsymbol{e}_2 \cdot \boldsymbol{e}_2 \end{pmatrix}$$

The first attempt to get realistic physics in $D=4$ used $E_8 \times E_8$ heterotic string: MSSM-like physics appear from compactification on Calabi–Yau manifolds!

• To get physics with Yang–Mills in spacetime we will need Dp-branes with $p \geqslant 6$, in fact here we use $p=6$.

Let's consider two D6-branes intersecting as follows.

$$\mathrm{D6}_1 \quad \begin{cases} x^+, x^-, x^2, x^3, x^4, x^6, x^8 & \text{Neumann boundary conditions} \\ x^5, x^7, x^9 \equiv 0 & \text{Dirichlet boundary conditions} \end{cases}$$

$$\mathrm{D6}_2 \quad \begin{cases} x^+, x^-, x^2, x^3, x^5, x^7, x^9 & \text{Neumann boundary conditions} \\ x^4, x^6, x^8 \equiv 0 & \text{Dirichlet boundary conditions} \end{cases}$$

These intersect where all D conditions are satisfied, i.e. for

$$x^4 = x^5 = x^6 = x^7 = x^8 = x^9 = 0.$$

This is a point on T^6, and no restriction at all in x^+, x^-, x^2 and x^3. This is 4-dimensional space-time.

We note that $T^6 = T^2 \times T^2 \times T^2$

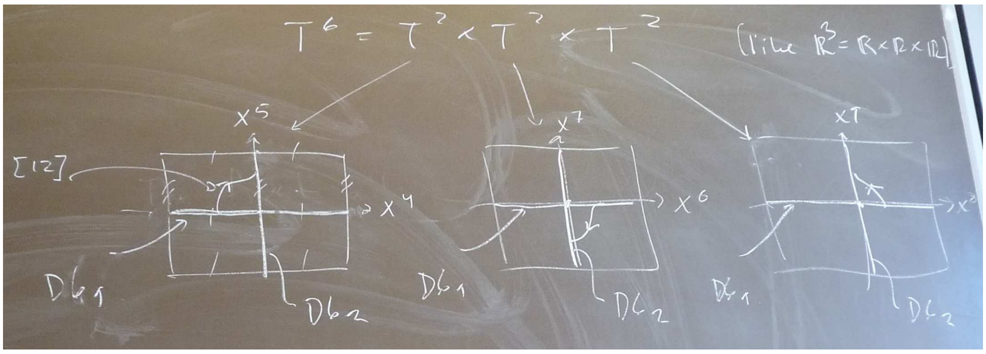

Figure 2.

Open string sectors $[ij]$, $i, j = 1, 2$.

Note: These open strings in sectors $[ij]$, $i \neq j$ have boundary conditions? Ex. x^5 in sector [12]: Dirichlet–Neumann. We have seen Dirichlet–Neumann before in

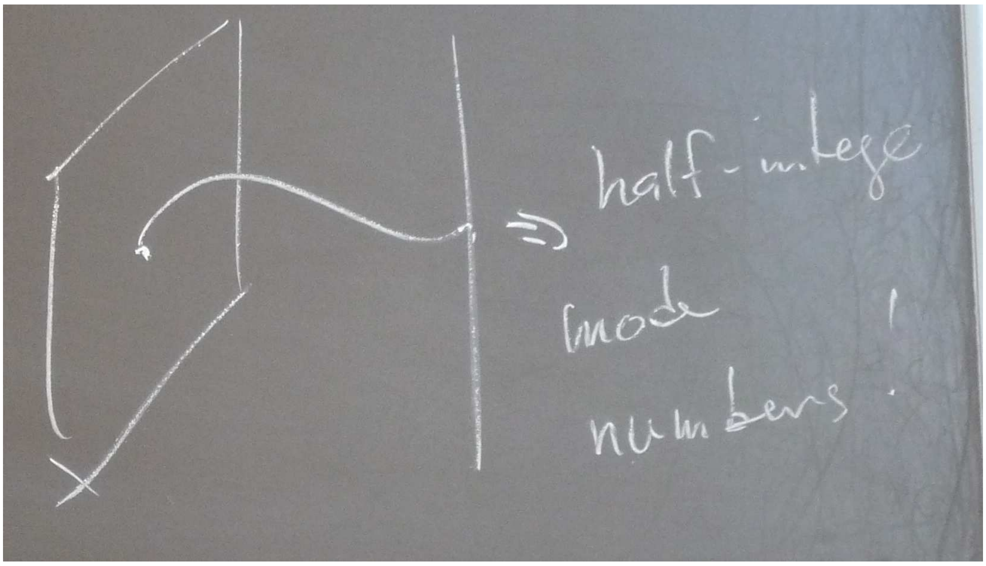

Figure 3. Half-integer mode numbers

Half integer mode numbers and no momentum. The open string is tied to the intersection!

So each intersection point gives a new sector of states.

Multiple intersections

On each T^2 in T^6.

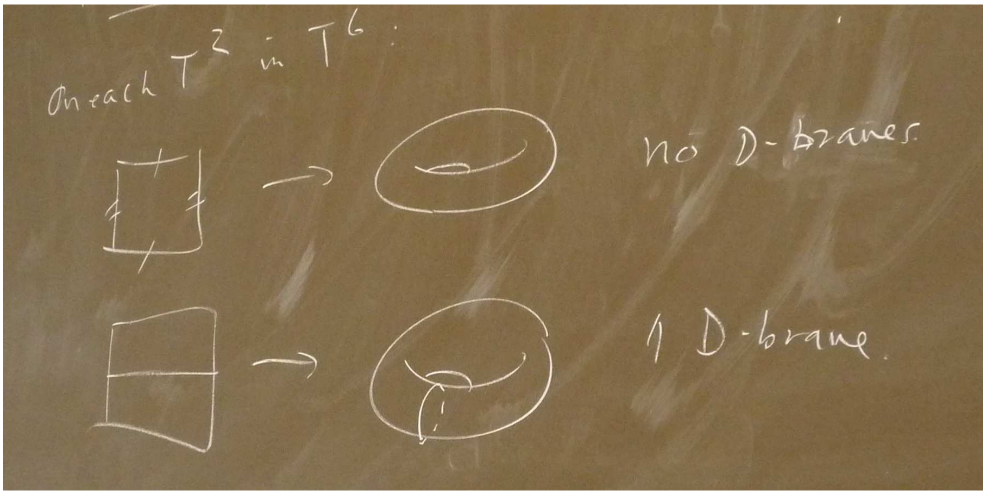

Figure 4.

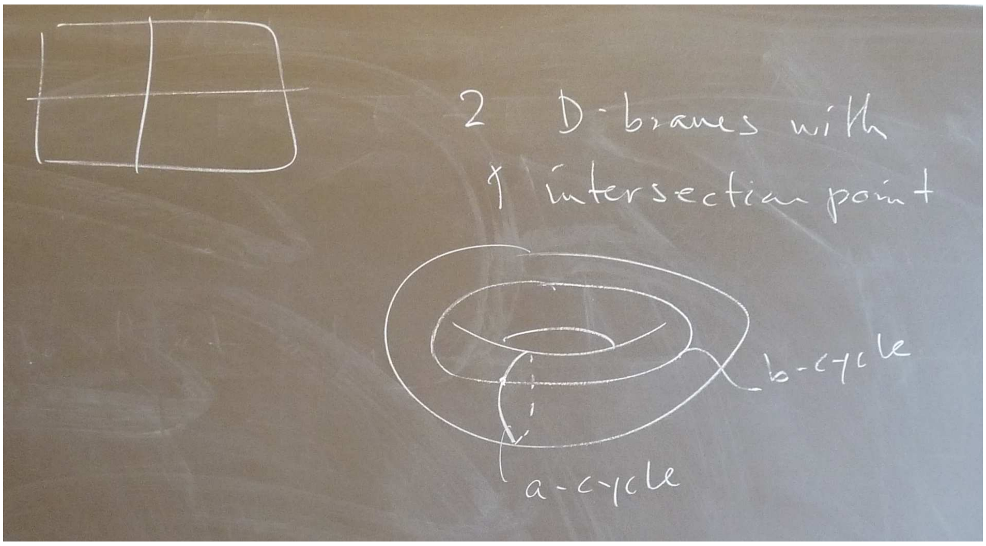

Figure 5.

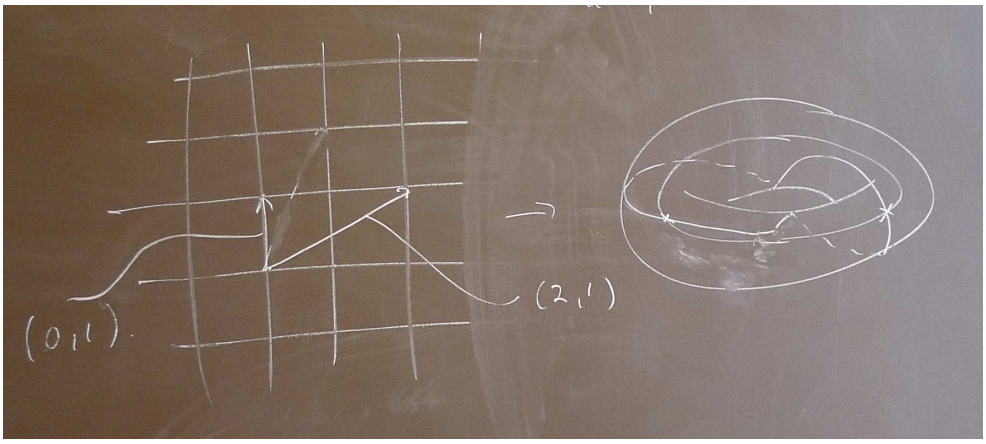

Figure 6.

Introduce $l=(m,n)$. Intersection number of the two vectors $l_1=(m_1,n_1)$ and $l_2=(m_2,n_2)$ is

$$\text{number}=\det(l_1,l_2)=\det\begin{pmatrix} m_1 & n_1 \\ m_2 & n_2 \end{pmatrix}=m_1n_2-n_1m_2$$

In the example: number $=2\times 1-1\times 0=2=I_{12}=-I_{21}$.

21.2: D-branes and Yang–Mills fields

An N-stack of D-branes $\Rightarrow \mathrm{U}(N)$ Yang–Mills theory on it.

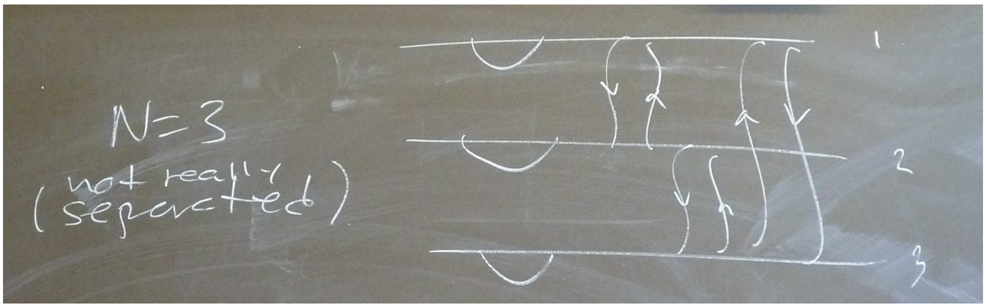

Figure 7. $N=3$ (not really separated)

$\Rightarrow$ 3 U(1) massless vectors from sectors $[i\,i]$ (no sum) plus 6 massless vectors from $[i\,j]$, $i\neq j$. States

$$\alpha_{-1}^{\mu}|i\,j\rangle\rightarrow A_{\mu}^{a}(T^{a})_{i}^{\;j}$$

where T^a are the generators of the group $\mathrm{U}(N)$.

Then the trace $\sum_i A_{\mu}^{a}(T^a)_i^i$ is special (the sum of the three U(1)'s above). In fact this U(1) does not interact with the other 8 vectors.

So

$$\mathrm{U}(N)=\mathrm{SU}(N)\times\mathrm{U}(1)$$

Example: The U(1):

$$\frac{1}{\sqrt{3}}\big(\alpha_{-1}^{\mu}|11\rangle+\alpha_{-1}^{\mu}|22\rangle+\alpha_{-1}^{\mu}|33\rangle\big)$$

Then the two orthogonal states are

1) $\alpha_{-1}^{\mu}|11\rangle+\alpha_{-1}^{\mu}|22\rangle-2\alpha_{-1}^{\mu}|33\rangle$

2) $-\alpha_{-1}^{\mu}|11\rangle+\alpha_{-1}^{\mu}|22\rangle$.

Related to diagonal Gell-Mann matrices.

Then to get SU(3) $\times$ SU(2) $\times$ U(1) we need one 3-stack: U(3) = SU(3) $\times$ U(1) and one 2-stack: U(2) = SU(2) $\times$ U(1). We seem to have one U(1) too many.

21.3: Open strings and Standard Model fermions

Recall the chirality property of the Standard Model fermions

$$\mathrm{repr}(f_L^{\dagger})\neq\mathrm{repr}(\bar{f}_L^{\dagger})$$

$f_L^\dagger$: left handed particle creation operators.

$\bar{f}_L^\dagger$: left-handed anti-particle creation operator.

Standard model:

$$3 \times \Big((3,2)_{1/6} + \big(\bar{3},1\big)_{-2/3} + \big(\bar{3},1\big)_{1/3} + (1,2)_{-1/2} + (1,1)_1 + (1,1)_0 \Big)$$

$$(\mathrm{SU}_C(3) \times \mathrm{SU}_W(2))_Y$$

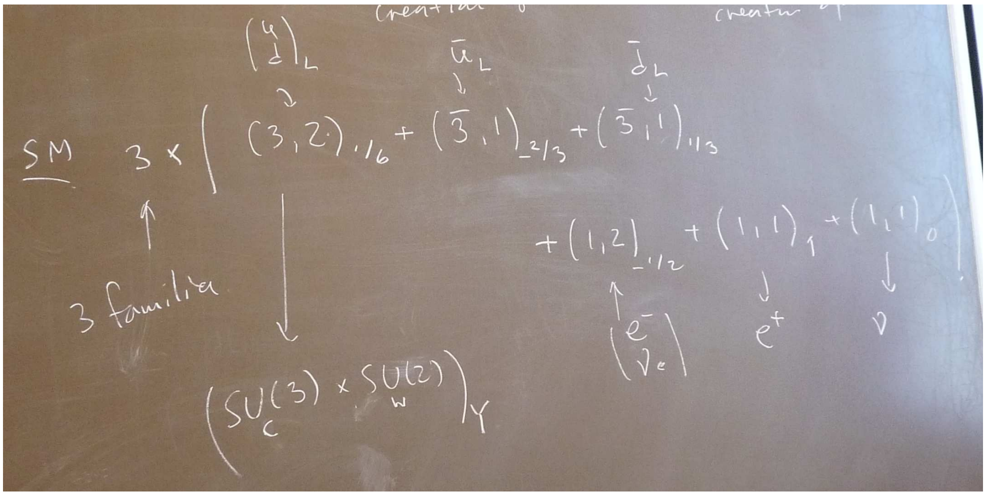

Figure 8.

Then we define, after SU(2) symmetry breaking

$$Q_{\mathrm{EM}} = Y + I_3$$

Can we reproduce these features of the standard model from some D6-brane configuration?

Introduce a colour 3-stack

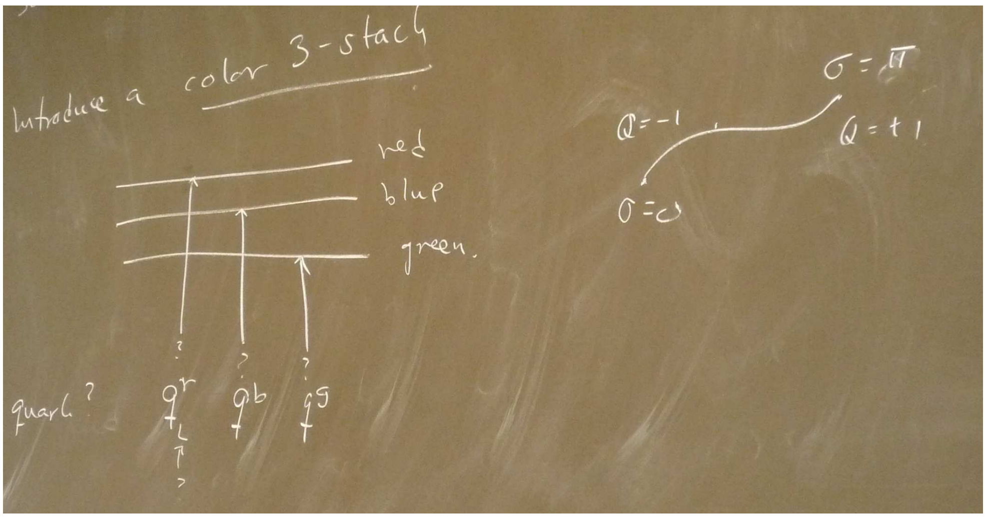

Figure 9.

So this way we get the 3 of the $(3,2)_{1/6}$. The other end of these open strings must end on a 2-stack.

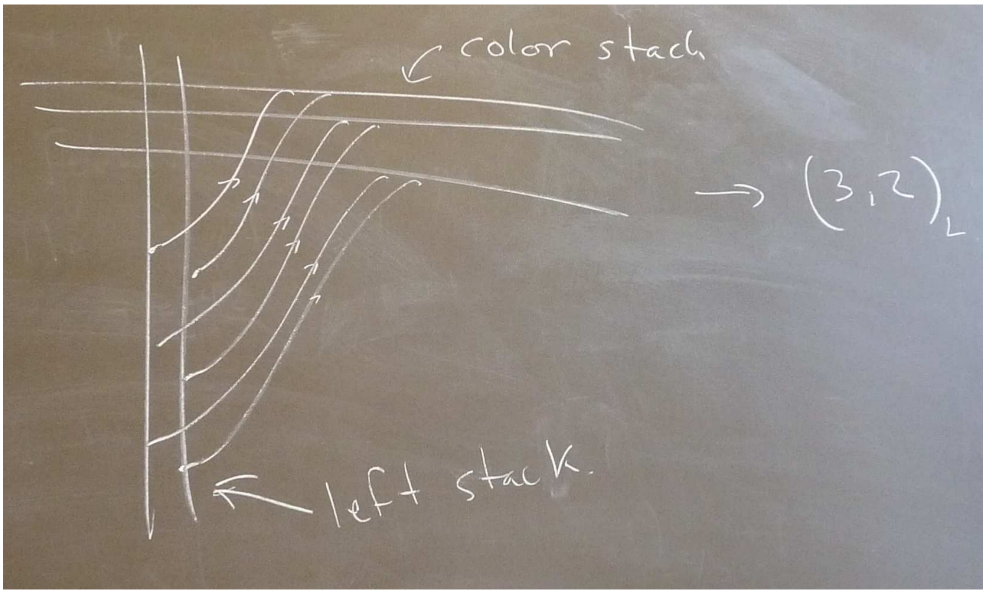

Figure 10. Antiparticles go in the opposite direction.

Where are the $(\bar{3},1)$ quarks?

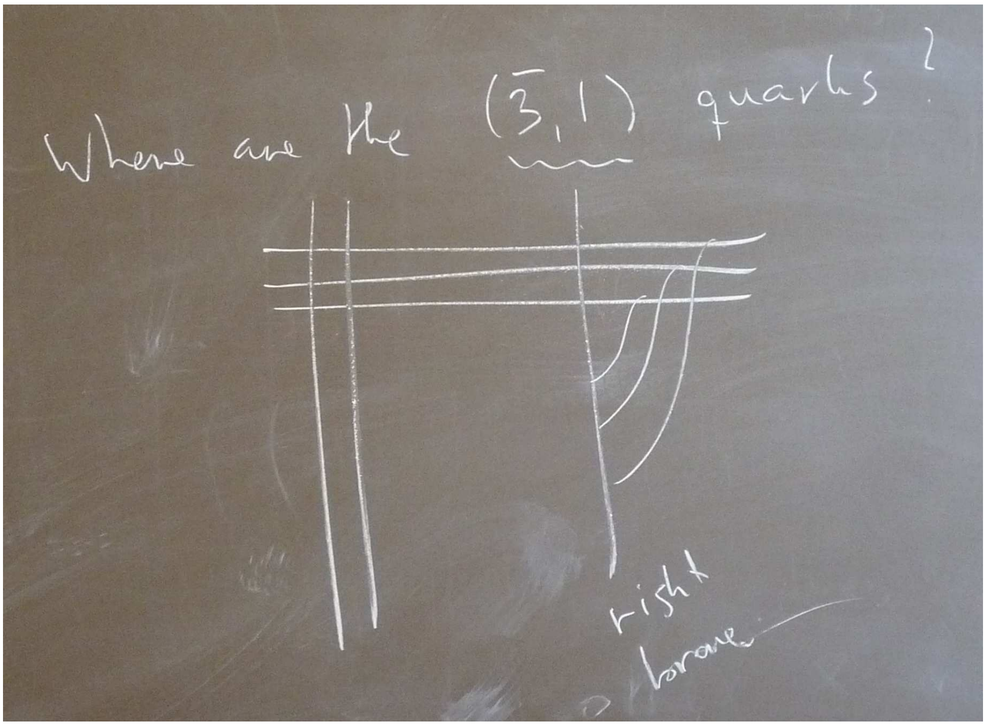

Figure 11.

Here: colour 3-stack: $l_1^{(1)}=(1,2), l_2^{(1)}=(1,-1), l_3^{(1)}=(1,-2)$, lower index: torus number, upper index: D6-brane number.

Left 2-stack: $l_1^{(2)}=(1,1), l_2^{(2)}=(1,-2), l_3^{(2)}=(-1,5)$.

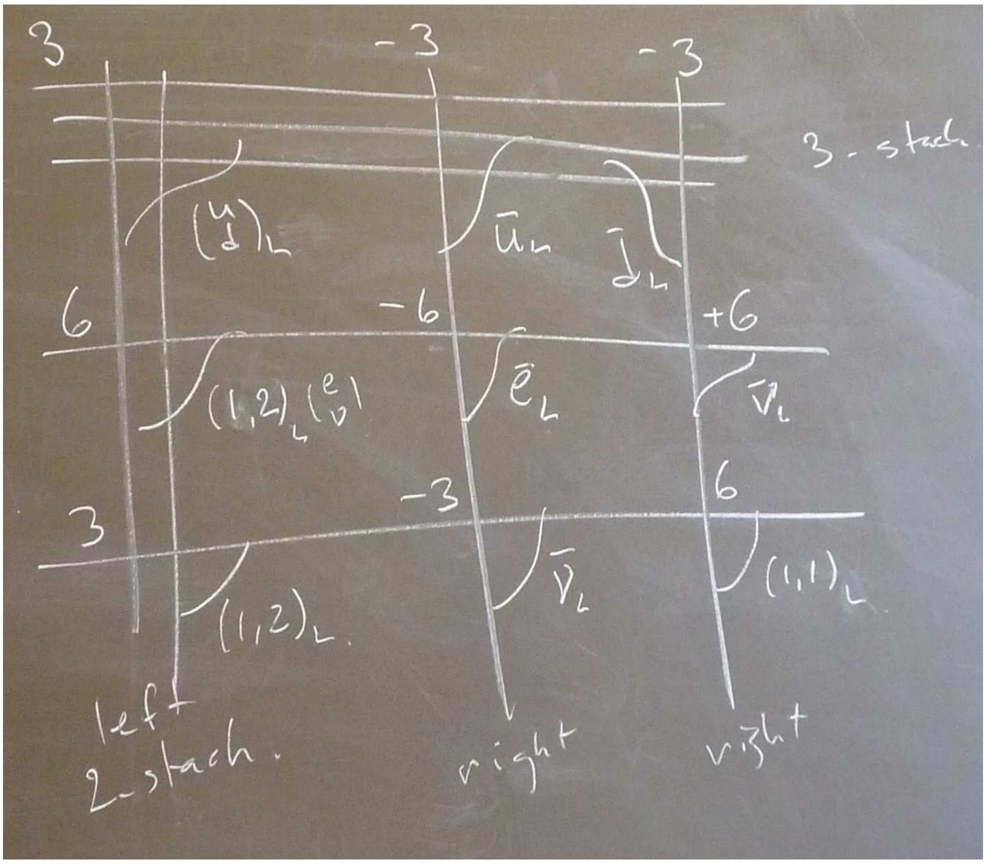

Figure 12.

This picture contains all particles in the three families but also some other particles.

Hypercharge: $Y = -\frac{1}{3}Q_1 - \frac{1}{2}Q_2 - Q_3 - Q_5$.

In order to get exactly the three standard model families this picture must be augmented with orientifold planes.