2009–03–17

Lecturer: Bengt E W Nilsson

Bengt strongly recommends the course book (Zwiebach, *A first course in String Theory*, second edition).

Home page can be reached from `http://fy.chalmers.se/~tfebn/`.

Send an email with name, national identification number and CTH/GU/...

Schedule: Monday 15, Tuesdays 15^{15}, (tomorrow Wednesday 10^{00}) and Thursdays 10^{00}.

Chapter 1: Brief introduction.

§ 1.1: Unification!

An example of unifications we have encountered in the physics education is the unification of electrics and magnetic phenomena into electromagnetism, and we get a theory for light for free.

Unification is a bout finding concepts and formulae with wider applicability, i.e. *reduction* of number of basic concepts and ideas. If you find this interesting, you can read Steven Weinberg's book, *Dreams of a Final Theory*.

Questions: What about the human mind? Here you can read Penrose, *The Emperor's New Mind*.

History of unification:

1687: Isaac Newton, *Philosophiæ Naturalis Principia Mathematica*. Mechanics on and off the earth.

1865: Maxwell: Maxwell's equations. Electrodynamics, magnetism, light unified.

1905: Einstein: Special relativity. (Knowledge of special relativity will be very useful.) Unifies space and time, and mechanics and electromagnetism. Along with space and time goes another unification: mass and energy.

1914: Einstein: General relativity. Unification of special relativity and gravity.

Further ideas:

$<$ 1915 Nordström: 5-dimensional electromagnetism $\rightarrow$ 4-dimensional electromagnetism + a scalar field. He tried to make this scalar field a theory of scalar gravity.

1919 Kaluza: 5-dimensional gravity $\rightarrow$ 4-dimensional gravity + electromagnetism (did not do it quite right).

1920 Klein realised that the above $\rightarrow$ 4-dimensional gravity + electromagnetism + scalar.

1930 Generalised to even more extra dimensions.

Almost found SU(2) gauge theory.

(Electromagnetism is a gauge theory based on U(1). You can think of U(1) as a circle, S^1; this will become clear later on. Yang-Mills (1954) is a gauge theory based on SU(2). You can think of SU(2) as S^3.)

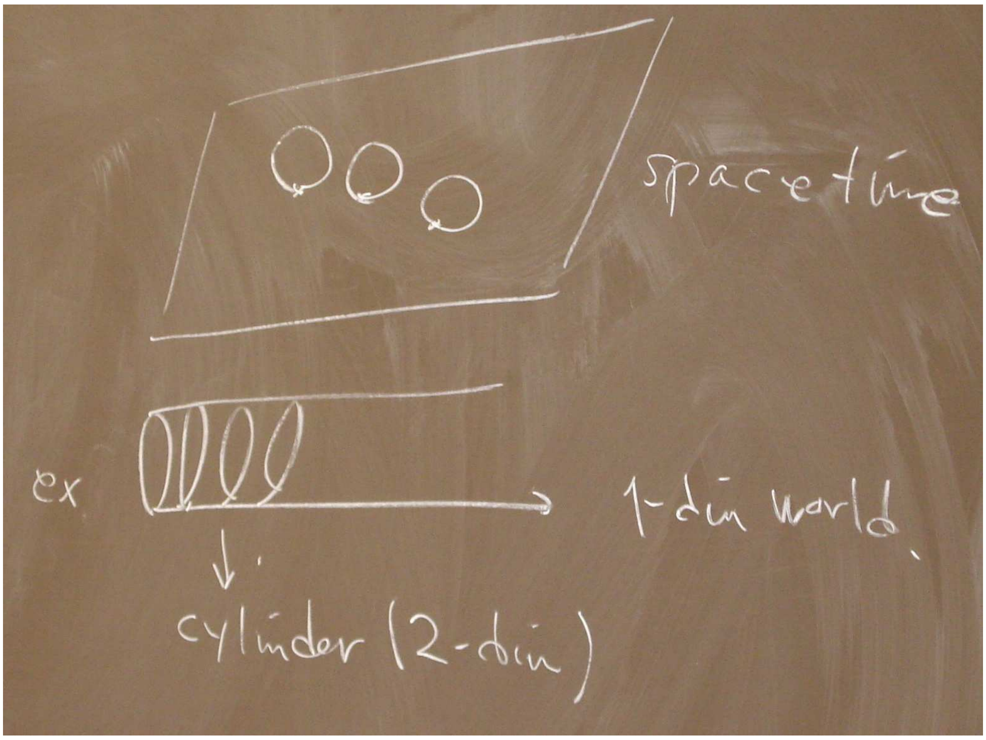

Figure 1. A circle in each point.

1968: Glashow, Weinberg, Salam: Electroweak theory. Contrary to the other ones, this is a well-established theory. This is based on SU(2) $\times$ U(1).

1970's: GUT-theories (not yet established). GUT stands for Grand Unified Theory. This should be a theory combining electromagnetism + weak interactions + strong interactions: the entire

Standard Model, preferably in one gauge group.

$$\left.\begin{array}{lcl} \text{EM} & \sim & \text{U}(1) \\ \text{weak} & \sim & \text{SU}(2) \\ \text{strong} & \sim & \text{SU}(3) \end{array}\right\} \Rightarrow ? \quad \begin{array}{l} \text{SU}(5), \text{E}_6, \text{SO}(10)\,\text{etc.} \\ \text{(Examples of GUT's)} \end{array}$$

Group theory. Rotation group:

$$\text{SO}(3) \sim \left(\begin{array}{ccc} \times & \times & \times \\ \times & \times & \times \\ \times & \times & \times \end{array}\right) = R, \quad R^T R = 1$$

Spin in three dimensions: SU(2). You need 2×2 complex matrices:

$$U = \left(\begin{array}{cc} \times & \times \\ \times & \times \end{array}\right), \quad U^\dagger U = 1$$

$$u = \mathrm{e}^{\mathrm{i}\alpha^i \sigma^i}$$

where σ^i are the Pauli matrices, satisfying $[\sigma^i, \sigma^j] = \mathrm{i}\, \varepsilon^{ijk}\, \sigma^k$. This is a Lie algebra. The α^i are three angles.

$$R^T R = 1 \Rightarrow \text{any dimension } n \text{ (real):} \quad \text{SO}(n)$$

$$U^\dagger U = 1 \Rightarrow \text{any dimension } n \text{ (complex):} \, \text{SU}(n).$$

The S in SU(n) tells us that $\det U = 1$.

Cartan: Classification:

$$\begin{array}{lcl} A_p & \sim & \text{SU}(\ ) \\ B_p & \sim & \text{SO}(\ )\,\text{in odd dimensions} \\ C_p & \sim & \text{SP}(\ ) \\ D_p & \sim & \text{SO}(\ )\,\text{in even dimensions} \end{array}$$

These are the classical matrix groups.

There are 5 exceptional cases: G_2, E_6, E_7, E_8, F_4.

Back to our discussion about unification.

1920's: Quantum Mechanics. Quantum Mechanics is a *framework*. For example: atoms. It is a theory where you apply electromagnetism. But you have to do it together with quantum mechanics. EM + QM $\Rightarrow$ QED (quantum electrodynamics, checked to at least eleven digits!)

't Hooft got Nobel prize for (see figure)

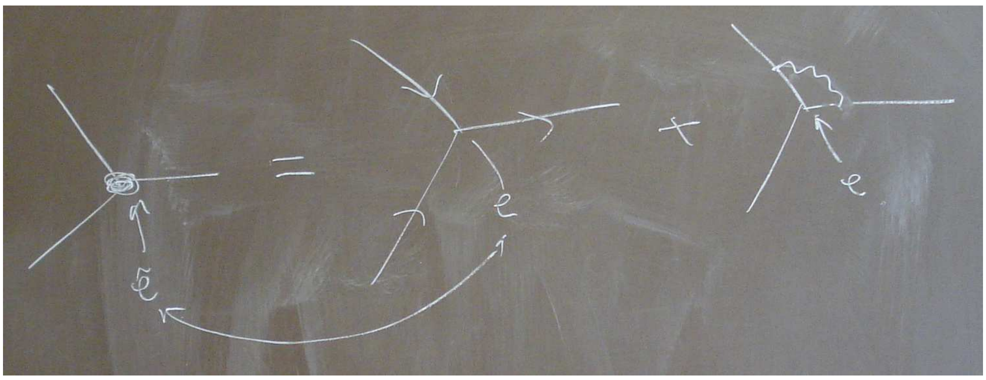

Figure 2. Measured charge $\tilde{e}$ and charge in the Lagrangian e.

Works great. Quantum field theory.

1970's: Gravity and quantum mechanics? Does not work at all.

Strings!

1980's: Many string theories, which got unified into M-theory. The many string theories live in 10 dimensions, whereas M-theory lives in 11 dimensions. The traditional point of view is that this is a theory in the same sense that electromagnetism is a theory, and that this would describe the entire evolution of the universe (including the Big Bang itself).

What is string-theory/M-theory?

There are two ways of looking at it. Either it is a TOE: a theory of everything. Or it could be a framework, similar to Quantum Mechanics.

From string theory we can derive — without putting it in — Einstein's equations.

Classical string + quantum mechanics ⇒ Quantum gravity + (if you choose the dimensions suitably) the Standard Model of particle physics.

§ 1.2: String theory as a unified theory of physics:

- All forces and particles are vibrational modes of a string.
- It is a quantum mechanical version of gravity, which is generic. In any version of string theory, there will always be a gravitational field. It "predicts" Einstein's General Relativity.
- String/M-theory is unique, in the sense that it has *no* free dimensionless parameters. Compare to the Standard Model — it has more than 20 unknown parameter: masses, charges, funny angles.

  The theory is unique, but there are at least 10^{500} solutions!
- It unifies particles, strings, etc *and* solitons.

§ 1.3: String theory and its verification!

See the home page.

At CERN (LHC):

- Higgs particle (does not say much about string theory)
- Supersymmetric particles. We might get dark matter as a bonus here.
- Extra dimensions. That would be a good indication of string theory.

What is the size of a string?

History: Prior to 1976 strings where used to describe hadrons.

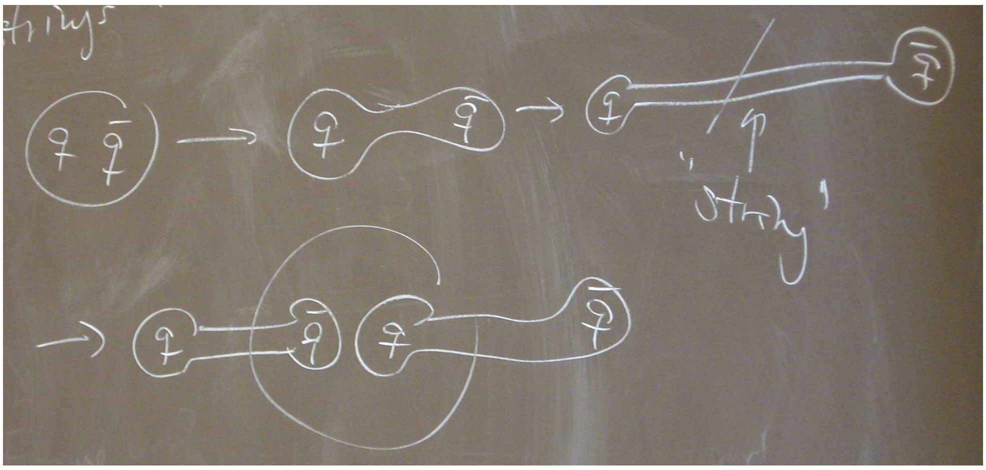

Figure 3. Pulling quarks apart.

In this picture the string length l_s would be on the order of $1\,\text{fm} = 10^{-15}\,\text{m}$.

1976: Hadronic string has a spin 2 massless particle! This is the graviton. In this picture $l_s \sim l_{\text{Planck}} \sim 10^{-35}$ m. The LHC tests sizes of 10^{-20} m, so we will never be able to get down to a such small l_s.

§ 1.4: Developments and outlook:

- What is string theory? Einstein's equations come from the equivalence principle. A corresponding principle behind string theory is not known.