## 2008–10–01

Electron rotating around a nucleus. It has a spin and an angular momentum $L$.

Spin-orbit coupling $\Rightarrow \alpha \boldsymbol{L}\cdot\boldsymbol{S}$.

$$\boldsymbol{B}_? = \frac{\boldsymbol{v}}{c}\times\boldsymbol{E} = \frac{\boldsymbol{v}}{c}\times\frac{Z\,e}{r^3}\,\boldsymbol{r} = -\frac{Z\,e}{m\,r^3\,c}\boldsymbol{r}\times\boldsymbol{p} = -\frac{Z\,e}{m\,r^3 c}\,\boldsymbol{L}$$

$$H_{\text{eff}} = -\,\mu\,\boldsymbol{B}_{\text{eff}}\cdot\boldsymbol{S} = \frac{1}{2}\cdot\frac{Z\,e^2}{m^2c^2r^3}\,\boldsymbol{L}\cdot\boldsymbol{S}$$

$$H = \alpha\,\boldsymbol{L}\cdot\boldsymbol{S}$$

Spin $1/2$, angular momentum $l=1$.

$$H = \frac{\alpha}{2}\big(J_{\text{tot}}^2 - L^2 - S^2\big), \quad 1\otimes\frac{1}{2} = \frac{3}{2}\otimes\frac{1}{2}$$

$$J_{\text{tot}} = \frac{3}{2}\!: \quad \frac{\alpha}{2}\left[\frac{3}{2}\cdot\frac{5}{2} - 1\cdot 2 - \frac{1}{2}\cdot\frac{3}{2}\right] = \frac{\alpha}{2}\left[\frac{15}{4} - \frac{8}{4} - 34\right] = \frac{?\alpha}{2}$$

$$J_{\text{tot}} = \frac{1}{2} \quad\Rightarrow\quad \frac{\alpha}{2}\left[\frac{3}{4} - 2 - \frac{3}{4}\right] = -\,\alpha.$$

Magnetic field $\boldsymbol{B}\cdot(\boldsymbol{L} + g_e\boldsymbol{S})$

Example

$$\frac{1}{2}\otimes\frac{1}{2} = 1\oplus 0$$

$$|j_1 j_2 j m\rangle = \sum_{m_1 m_2} |j_1 j_2 m_1 m_2\rangle\langle j_1 j_2 m_1 m_2|j_1 j_2 j m\rangle$$

$$|j m\rangle = \sum_{m_1 m_2} \langle m_1 m_2|j m\rangle \quad |m_1 m_2\rangle$$

Singlet

$$|0,0\rangle = \langle\frac{1}{2}, -\frac{1}{2}|0,0\rangle|\frac{1}{2}, -\frac{1}{2}\rangle + \langle -\frac{1}{2}, \frac{1}{2}|0,0\rangle\,| -\frac{1}{2}, \frac{1}{2}\rangle = \frac{1}{\sqrt{2}}\left(|\frac{1}{2}, -\frac{1}{2}\rangle - | -\frac{1}{2}, \frac{1}{2}\rangle\right)$$

...

$$r\,Y_1^0 = \sqrt{\frac{3}{4\pi}}\,z = T_0^1$$

$$r\,Y_1^{+1} = -\sqrt{\frac{3}{4\pi}}\,(x + \mathrm{i}y)/\sqrt{2} = T_{+1}^1$$

$$r\,Y_1^{-1} = \sqrt{\frac{3}{4\pi}}(x - \mathrm{i}y)/\sqrt{2} = T_{-1}^1$$

Define irreducible tensor operators

$$\mathcal{D}^\dagger(R)\, T^j_m\, \mathcal{D}(R) = \sum_{m'} \mathcal{D}^j_{m\,m'}(R)\, T^j_{m'}$$

$$|jm\rangle' = \mathcal{D}^i_{m\,m'}(R)\, |jm\rangle$$

$$\left(T^j_m\right)' = \mathcal{D}^j_{m\,m'}(R)\, T^j_m$$

$$|j_1\, m_1\rangle \otimes |j_2 m_2\rangle = \mathcal{D}^{j_1}_{m_1 m_1'}(R)\, \mathcal{D}^{j_2}_{m_2 m_2'}(r) |j_1 m_1'\rangle \otimes |j_2 m_2'\rangle$$

$$T^j_{m_1} |j_2 m_2\rangle = \mathcal{D}^{j_1}_{m_1 m_1'}(R)\, \mathcal{D}^{j_2}_{m_2 m_2'}(R)\, T^{j_1}_{m_1'} |j_2\, m_2'\rangle$$

$$|j_1 j_2 jm\rangle = \sum_{j,m} |j_1 j_2 m_1 m_2\rangle \langle j_1 j_2 m_1 m_2 | j_1 j_2 jm\rangle$$

$$|j_1 m_1\rangle \otimes |j_2 m_2\rangle = \sum_{j,m} |j_1 j_2 jm\rangle \langle j_1 j_2 jm | j_1 j_2 m_1 m_2\rangle$$

Wigner Eckart Theorem:

$$T^{j_1}_{m_1} |j_2 m_2\rangle = \sum_{jm} N_{j,j_1,j_2} |jm\rangle \langle j_1 j_2 jm | j_1 j_2 m_1 m_2\rangle$$

Dipole radiation $\rightarrow$ transitions $j \rightarrow j \pm 1$.

Quadrupole $T^2_m$

$$\langle jm | T^2_0 | j' m'\rangle = 0$$

unless $|j - j'| \leqslant 2$, $j' = j \pm 2, j,\ m = m'$.

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What is $T^2_0$?

$$T^1_1 = -\,(x + \mathrm{i}\, y)\sqrt{2}$$

$$T^1_0 = z$$

$$T^1_{-1} = (x - \mathrm{i} y)\sqrt{2}$$

$$T^2_0 = \frac{1}{\sqrt{6}} \big(T^1_1\, T^1_{-1}\big) + \sqrt{\frac{2}{3}} \big(T^1_0 T^1_0\big) + \frac{1}{\sqrt{6}} \big(T^1_{-1} T^1_1\big)$$