2008–09–10

Ling Bao.

1.4 a) Prove $\mathrm{tr}(XY) = \mathrm{tr}(YX)$.

$$\mathrm{tr}(XY) \equiv \sum_{\alpha'} \langle\alpha'|XY|\alpha'\rangle$$

$$\mathrm{tr}(XY) = \sum_{\alpha'} \sum_{\alpha''} \langle\alpha'|X|\alpha''\rangle\langle\alpha''|Y|\alpha'\rangle = \sum_{\alpha''} \langle\alpha''|YX|\alpha''\rangle = \mathrm{tr}(XY).$$

b) Prove $(XY)^\dagger = Y^\dagger X^\dagger$

$$(XY)\,|\alpha\rangle \quad \leftrightarrow \quad \langle\alpha|(XY)^\dagger$$

$$X(Y|\alpha\rangle) \quad \leftrightarrow \quad (Y|\alpha\rangle)^\dagger X^\dagger \quad \longleftrightarrow \quad \langle\alpha|Y^\dagger X^\dagger$$

c) $\exp(\mathrm{i}\, f(A))$...

Ling Bao is writing so fast on the whiteboard that I cannot both think about what she is saying and writing down what she is writing, at the same time. But she follows the solutions available on the course web page quite closely.