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Last time: Electromagnetism and strong interactions.

$$\mathcal{L} = -\frac{1}{4} F_{\mu\nu}^{(\gamma)} F^{(\gamma)\mu\nu} + \frac{1}{8\,g_c^2} \operatorname{tr}\Big(G_{\mu\nu}^{(c)} G^{(c)\mu\nu}\Big) + \sum_f \bar{q}_f \Big(\mathrm{i}\not{D}_f - m_f\Big) q_f +$$

$$+ \sum_l \left[\bar{\psi}\left(\mathrm{i}\not{D}_l - m_l\right)\psi_l + \bar{\nu}_l\big(\mathrm{i}\not{\partial} - m?\big)\nu_l\right] + \text{more}$$

Flavours $f = u,\, d,\, s,\, c,\, t,\, b$. Charged leptons $l = e,\, \mu,\, \tau$. Dirac spinors: $\psi,\, \nu,\, q$. The index (c) above stands for colour.

$$\not{D}_f = \begin{cases} \gamma^\mu\Big(\partial_\mu - \frac{2}{3}\,\mathrm{i}\,e\,A_\mu^{(\gamma)} + A_\mu^{(c)}\Big) & \text{for } f = u, c, t \\ \gamma^\mu\Big(\partial_\mu + \frac{1}{3}\,\mathrm{i}\,e\,A_\mu^{(\gamma)} + A_\mu^{(c)}\Big) & \text{for } f = d, s, b \end{cases}$$

$$\not{D}_l = \gamma^\mu\Big(\partial_\mu + \mathrm{i}\,e\,A_\mu^{(\gamma)}\Big) \quad \text{for } l = e,\, \mu,\, \tau$$

$e =$ electron charge; note $e < 0$ in the notation used by Peskin&Schroeder.

Strong interactions: asymptotic freedom. Effective coupling scale dependent:

$$g_c^2(q) = \frac{g_c^2(\mu)}{1 + \frac{g_c^2(\mu)}{4\pi^2}\Big(\frac{11}{3}N - \frac{2}{3}n_f\Big)\ln\frac{q^2}{\mu^2}}$$

$N = 3$, the 3 of SU(3). $n_f = 6$.

Confinement hypothesis: Hadrons are colour singlets. Baryons: qqq, mesons: $q\,\bar{q}$. $\varepsilon^{abc} q_a\, q_b\, q_c$ and $\bar{q}^{\,a} q_a$.

In addition there is weak interaction.

Examples: $\mu \to e + \bar{\nu}_e + \nu_\mu$. $n \to p + e + \bar{\nu}_e$ or $d \to u + e + \bar{e}$ [sic].

$k^- \to \mu^- + \bar{\nu}_\mu$, $k^- \to \pi^- + \pi^0$. k^- is $\bar{u}s$. $s \to u + \mu + \bar{\nu}_\mu$. π^- is $\bar{u}d$. $s \to u + d + \bar{u}$.

Phenomenological model.

Fermi's current current model.

$$\mathcal{H}_{\mathrm{WI}} = \frac{G_F}{\sqrt{2}}\, j_\lambda^\dagger j^\lambda, \quad G_F = 1.17 \times 10^{-5}\,\mathrm{GeV}^{-2}$$

$$j_\lambda = \sum_{l=e,\mu} \bar{\psi}_\lambda \gamma_\lambda\big(1 - \gamma^5\big)\nu_l + \big(c(\theta_C)\bar{\psi}_d + S(\theta_C)\bar{\psi}_s\big)\gamma_\lambda(1 - \gamma_5)\psi_u + \ldots$$

$$\theta_C = 0.24$$

In the Standard Model these processes are described by Feynman graph:

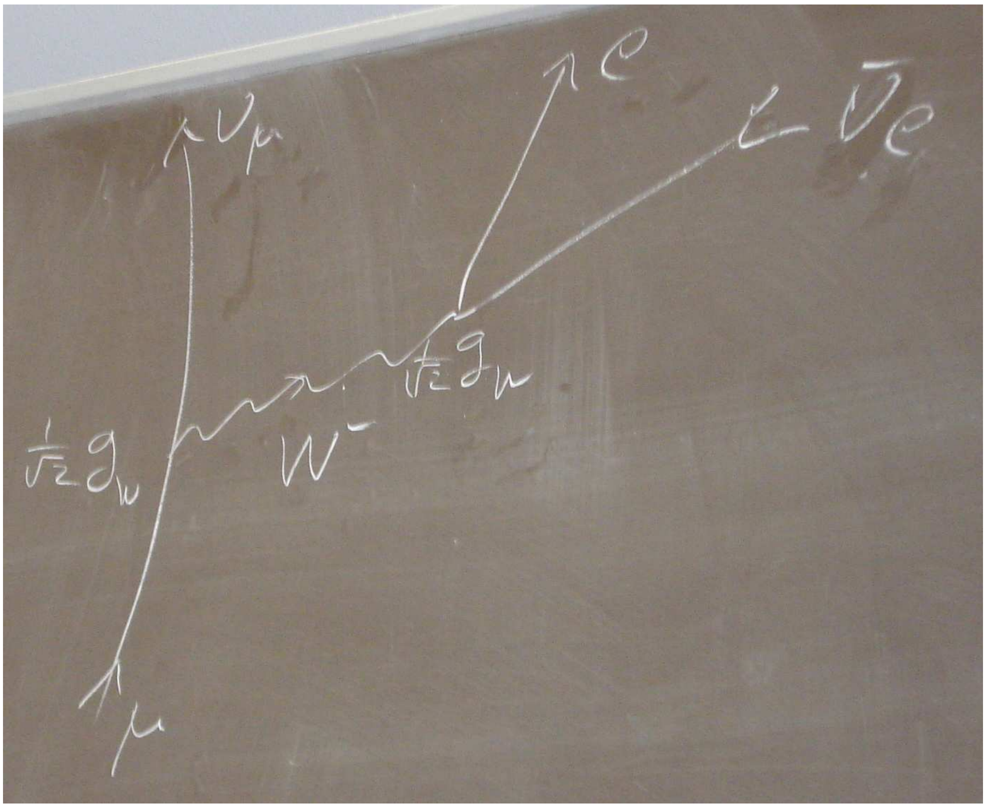

Figure 1.

$$\frac{G_F}{\sqrt{2}} = \frac{g_W^2}{8\,M_W^2} = \frac{e^2}{8\,M_W^2 S^2(\theta_W)}, \qquad M_W^2 = 80\,\text{GeV}, S^2(\theta_W) = 0.22$$

$$\frac{1}{2}\big(1 - \gamma^5\big) \text{ projects } \psi_D \text{ onto } \psi_L, \text{ upper two components of } \psi_D.$$

For massless fermions ψ_L describes helicity $-\frac{1}{2}$.

Building standard model

Fermions divided into three families.

					Masses	in	GeV	
1	u	d	e	ν_e	0.004	0.007	0.0005	
2	c	s	μ	ν_μ	1.5	0.2	0.1	10^{-13} (relative to ν_e)
3	t	b	τ	ν_τ	175	4.5	1.8	$10^{-11.5}$

First, focus on the first generation.

Divide all fermions into left and right components:

$$\psi_D = \begin{pmatrix} \psi_L \\ \psi_R \end{pmatrix}$$

ψ_R can in principle be expressed as a charge conjugated ψ_L. $\psi_R = \mathrm{i}\,\sigma^2\,\psi_L'$ (see problem 3.4c), but I will not do this.

Then, left handed fermions are recombined into doublets.

First generation fermions:

$$\begin{pmatrix} u_L \\ d_L \end{pmatrix} = q_L \quad u_R \quad d_R \quad \begin{pmatrix} \nu_{eL} \\ e_L \end{pmatrix} = \psi_L \quad \nu_{eR} \quad e_R$$

$\mathrm{U(1)_{EM}}$ is replaced by $(\mathrm{SU(2)} \times \mathrm{U(1)})_{W+\mathrm{EM}}$.

$$\mathcal{L}_0 = \bar{q}_L \, \mathrm{i} \not{D} \, q_L + \bar{u}_R \mathrm{i} \not{D} \, u_R + \bar{d}_R \, \mathrm{i} \not{D} \, d_R + \psi_L \mathrm{i} \not{\partial} \psi_L + \bar{\nu}_{eR} \, \mathrm{i} \not{\partial} \nu_{eR} + \bar{e}_R \, \mathrm{i} \not{\partial} \, e_R$$

$$\not{D} = \gamma^\mu \Big(\partial_\mu + A_\mu^{(c)} \Big)$$

Global $\mathrm{SU(2)} \times U(1)$ symmetry of $\mathcal{L}_0$ to be gauged.

$$\begin{cases} \delta q_L = \left(\frac{\mathrm{i}}{2} \, g \, \boldsymbol{\tau} \cdot \boldsymbol{\Lambda} + \frac{\mathrm{i}}{6} \, g' \, \Lambda \right) q_L \\ \delta u_R = \frac{2}{3} \, \mathrm{i} \, g' \, \Lambda \, u_R \\ \delta d_R = -\frac{1}{3} \, \mathrm{i} \, g' \, \Lambda \, d_R \\ \delta \psi_L = \left(\frac{\mathrm{i}}{2} g \, \boldsymbol{\tau} \cdot \boldsymbol{\Lambda} - \frac{1}{2} \, \mathrm{i} \, g' \, \Lambda \right) \psi_L \\ \delta u_L = 0 \\ \delta e_R = - \, \mathrm{i} \, g' \, e_R \end{cases}$$

Note $\mathrm{SU(2)} \times U(1)$ does not interfere with $\mathrm{SU(2)}_c$. It is now straight forward to gauge this $\mathrm{SU(2)} \times U(1)$ symmetry and write down most general renormalizable $\mathrm{SU(3)} \times \mathrm{SU(2)} \times \mathrm{U(1)}$ gauge invariant Lagrangian $\mathcal{L}$.

Note there is only one mass term:

$$\frac{\mathrm{i}}{2} \, M_\nu \Big(\nu_R^T \sigma^2 \nu_R - \nu_R^\dagger \sigma^2 \nu_R^* \Big)$$

Majorana mass term for right-handed ν (cf. problem 3.4 b).

Other mass terms are not $\mathrm{SU(2)} \times \mathrm{U(1)}$ invariant, because charges don't ad up to zero. (But m_ν should be the smallest mass?)

To give masses to fermions and W, Z, use Higgs mechanism. Introduce scalar field:

$$\phi = \begin{pmatrix} \phi^0 \\ \phi^- \end{pmatrix}, \quad \delta \phi = \left(\frac{\mathrm{i}}{2} \, g \, \boldsymbol{\tau} \cdot \boldsymbol{\Lambda} - \frac{\mathrm{i}}{2} \, g' \, \Lambda \right) \phi$$

$$\tilde{\phi} \equiv - \, \mathrm{i} \tau_2 \phi^*: \quad \delta \tilde{\phi} = \left(\frac{\mathrm{i}}{2} \, g \, \boldsymbol{\tau} \cdot \boldsymbol{\Lambda} + \frac{\mathrm{i}}{2} \, g' \, \Lambda \right) \tilde{\phi}$$

After gauging:

$$\mathcal{L} = \mathcal{L}_A + \mathcal{L}_\psi + \mathcal{L}_\phi = \mathcal{L}_A + \mathcal{L}_{\psi k} + \mathcal{L}_{\psi Y} + \mathcal{L}_{\phi k} + \mathcal{L}_{\phi \nu}$$