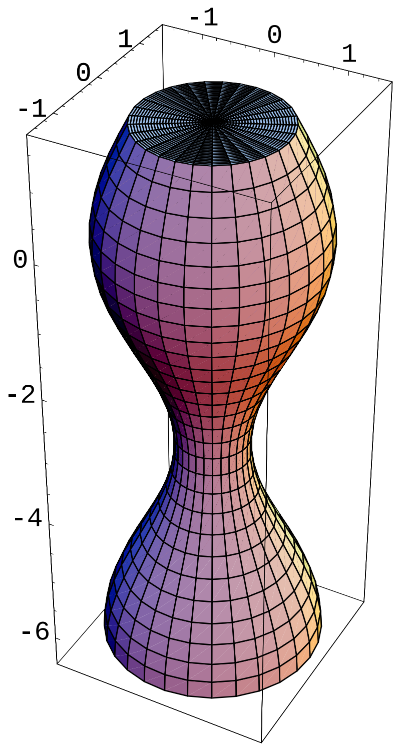
## 2006–05–02

### Reguljär precessionsrörelse (gyroskop)

Betrakta endast kroppar med rotationssymmetri. (Se figur 1).




**Figur 1.**

$$\mathcal{I} = \begin{pmatrix} I_0 & 0 & 0 \\ 0 & I_0 & 0 \\ 0 & 0 & I \end{pmatrix}$$

(Det viktiga här är att tröghetsmatrisen $\mathcal{I}$ har två lika egenvärden, ⟨fig40⟩)

⟨fig41⟩ $\boldsymbol{\nu}$ är **spinn**, $\boldsymbol{\Omega}$ är **precession**. "Läs gärna hela kapitel 7.11".

$$\dot{\boldsymbol{L}} = \boldsymbol{M} \quad \text{med } \dot{\boldsymbol{L}} = \frac{\mathrm{d}}{\mathrm{d}t}(\mathcal{I}\boldsymbol{\omega})$$

$\mathcal{I}$ är konstant endast i ett kroppsfixt system.

"$\theta = 90°$": ⟨fig42⟩

$$\boldsymbol{\omega} = \boldsymbol{\nu} + \boldsymbol{\Omega} = \nu\,\boldsymbol{e}_x + \Omega\,\boldsymbol{e}_z$$

$$\boldsymbol{L} = I\,\nu\,\boldsymbol{e}_x + I_0\,\Omega\,\boldsymbol{e}_z$$

$$\dot{\boldsymbol{L}} = \boldsymbol{\Omega} \times \boldsymbol{L} \quad (\dot{\boldsymbol{e}}_x = \boldsymbol{\Omega} \times \boldsymbol{e}_x = \Omega\;\boldsymbol{e}_y)$$

$$= I\,\nu\,\Omega\,\boldsymbol{e}_y$$

Skall vara lika med momentet $\boldsymbol{M} = m\,g\,d\,\boldsymbol{e}_y$.

Reguljär precession med $\theta = 90°$:

$$I\,\nu\,\Omega = m\,g\,d \quad \Longrightarrow \quad \Omega = \frac{m\,g\,d}{I\,\nu}$$

⟨fig43⟩

$$\boldsymbol{L} = \mathcal{I}\boldsymbol{\omega}$$

$$\boldsymbol{\omega} = \nu\,\boldsymbol{e}_\zeta + \Omega\,\boldsymbol{e}_z = \nu\,\boldsymbol{e}_\zeta + \Omega(\cos\theta\,\boldsymbol{e}_\zeta - \sin\theta\,\boldsymbol{e}_\xi) = (\nu + \Omega\cos\theta)\,\boldsymbol{e}_\zeta - \Omega\sin\theta\,\boldsymbol{e}_\xi$$

$$\boldsymbol{L} = I\,\left(\nu + \Omega\cos\theta\right)\boldsymbol{e}_\zeta - I_0\,\Omega\sin\theta\,\boldsymbol{e}_\xi$$

$$[\boldsymbol{e}_z = \boldsymbol{e}_\zeta\cos\theta - \boldsymbol{e}_\xi\sin\theta]$$

Eftersom kroppen är rotationssymmetrisk behöver vi inte låta $\xi$- och $\eta$-axlarna spinna med kroppen.

$$\dot{\boldsymbol{L}} = \boldsymbol{\Omega} \times \boldsymbol{L} = \Omega(\boldsymbol{e}_\zeta\cos\theta - \boldsymbol{e}_\xi\sin\theta) \times (I\,(\nu + \Omega\cos\theta)\,\boldsymbol{e}_\zeta - I_0\,\Omega\sin\theta\,\boldsymbol{e}_\xi)$$

Grafiskt: ⟨fig44⟩

$$= \Omega(-I_0\,\Omega\sin\theta\cos\theta\,\boldsymbol{e}_\eta + I(\nu + \Omega\cos\theta)\sin\theta\,\boldsymbol{e}_\eta) = \Omega\sin\theta\,\boldsymbol{e}_\eta\,[I\,\nu + (I - I_0)\Omega\cos\theta]$$

$$\boldsymbol{M} = m\,g\,d\sin\theta\,\boldsymbol{e}_\eta$$

$$m\,g\,d = \Omega(I\,\nu + (I - I_0)\,\Omega\cos\theta)$$

**Precession utan moment!**

Då måste $\dot{\boldsymbol{L}} = \boldsymbol{0}$, OK om $\boldsymbol{L} = L\,\boldsymbol{e}_z$.

$$I\nu + (I - I_0)\,\Omega\cos\theta = 0$$

$$\Omega = \frac{I\,\nu}{(I_0 - I)\cos\theta}$$

Låt $\nu > 0$, $\cos\theta > 0$

**Jorden** ⟨fig45⟩

$\theta \ll 1 \Longrightarrow \cos\theta \approx 1$.

$$\Omega \approx \frac{I}{I_0 - I}\nu = -\frac{1}{1 - \frac{I_0}{I}}\nu \approx -300\,\nu$$

⟨fig46⟩

$$\boldsymbol{\omega} \approx \frac{2\pi}{1\,\mathrm{dygn}}$$

$\boldsymbol{\nu}$ svarar mot ungefär 1 varv på 300 dagar.