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Lecturer: Gabriele Ferretti

MSSM: The supersymmetry breaking is done via soft terms. Electroweak symmetry breaking ($\sim 100\,\text{GeV}$) must be included.

$$H_u = \begin{pmatrix} H^+ \\ H^0 \end{pmatrix}, \quad H_d = \begin{pmatrix} H^0 \\ H^- \end{pmatrix}$$

$$\langle H_u \rangle = \begin{pmatrix} 0 \\ v \sin\beta \end{pmatrix}, \quad \langle H_d \rangle = \begin{pmatrix} v\cos\beta \\ 0 \end{pmatrix}$$

$\tan\beta$ is a parameter of the MSSM.

How? V_{Higgs}?

$\text{SU}(3) \times \text{SU}(2) \times \text{U}(1)_Y \rightarrow \text{SU}(3) \times \text{U}(1)_{\text{em}}$.

When looking at the spectrum all particles with the same Lorentz (spin) $\times$ SU(3) $\times$ $\text{U}(1)_{\text{em}}$ will mix.

After electroweak symmetry breaking: W^+, W^-, Z^0.

			SU(3)	U(1)	spin
W^+	$\rightarrow$	$\tilde{W}^+$	/	$+1$	$\frac{1}{2}$
W^-	$\rightarrow$	$\tilde{W}^-$	/	-1	$\frac{1}{2}$
Z^0	$\rightarrow$	$\tilde{Z}^0$	/	0	$\frac{1}{2}$
γ	$\rightarrow$	$\tilde{\gamma}$	/	0	$\frac{1}{2}$
		$\tilde{H}^\pm$	/	$\pm$	$\frac{1}{2}$
		$\tilde{H}$	/	0	$\frac{1}{2}$

Electron/muon/tau? We forgot about R-parity:

When looking at the spectrum all particles with the same Lorentz (spin) $\times$ SU(3) $\times$ U(1)$_{\text{em}}$ $\times$ R-parity will mix.

Initially $\tilde{W}^+\tilde{H}^+_{(u)} \longrightarrow 2$ charginos $\tilde{C}_i$.

$\tilde{W}^-\tilde{H}^- \rightarrow 2$ (anti-) charginos.

$\tilde{Z}^0, \tilde{\gamma}, \tilde{H}^0_u, \tilde{H}^0_d \rightarrow 4$ neutralinos $\tilde{N}_i$.

$$V_{\text{Higgs}}$$

Scalar potential $V_{\text{MSSM}}(QudLeH_uH_d) = V_D + V_F + V_S$ (soft). ϕ_i scalars:

$$V_F = \left|\frac{\partial W}{\partial \phi_i}\right|^2$$

From the off-shell Lagrangian:

$$F_i^\dagger F_i + \frac{\partial W}{\partial \phi_i} F_i + \frac{\partial W^\dagger}{\partial \phi_i^\dagger} F_i^\dagger$$

Solve for F_i:

$$F_i = -\frac{2W^+}{\partial \phi_i}$$

Off-shell $\frac{1}{2}(D^a)^2 + g\, D^a \phi_i^+ T^{a\,i}_j \phi^j$. Solve for $D^a = -g\phi^\dagger + a\phi$

$$V_D = \frac{g^2}{2}\left(\phi^\dagger T^a \phi\right)^2$$

$$V_{\text{Higgs}}, V_D = \frac{g^2 + g'^2}{8}\left(|H_u^0|^2 - |H_d^0|^2\right)^2$$

$$T_i \rightarrow \begin{matrix} \sigma^1 \\ \sigma^2 \\ \sigma^3 \end{matrix}$$

$$\phi \equiv H_u = \begin{pmatrix} 0 \\ u \end{pmatrix}$$

$$\phi^\dagger \sigma^1 \phi = \phi^\dagger \sigma^2 \phi = 0$$

$$V_F = \left|\frac{\partial W}{\partial H_u}\right|^2 + \left|\frac{\partial W}{\partial H_d}\right|^2$$

$H^{\pm} = 0$.

$$V_{\text{soft}} = m_{H_u}^2 |H_u^0|^2 + m_d^2 |H_d^0|^2 - b\, H_u^0 H_d^0 - b^* H_u^{0*} H_d^{0*}$$

$$V_{\text{Higgs}} = V_D + V_F + V_{\text{soft}}$$

Note that $m_{H_u}^2$ could be negative.

b can be made real. $\Rightarrow$ There is no CP violation in the Higgs sector of the MSSM. We will look for a minimum of V.

$$x = |H_u^0|, \quad y = |H_d^0|$$

$$V(x,y) = \big(|M|^2 + m_{H_u}^2\big) x^2 + \big(|M|^2 + m_{H_d}^2\big) y^2 - 2\, b\, x\, y + \frac{1}{8}\big(g^2 + g'^2\big)(x^2 - y^2)^2$$

Not a Mexican hat.

Is V stable?

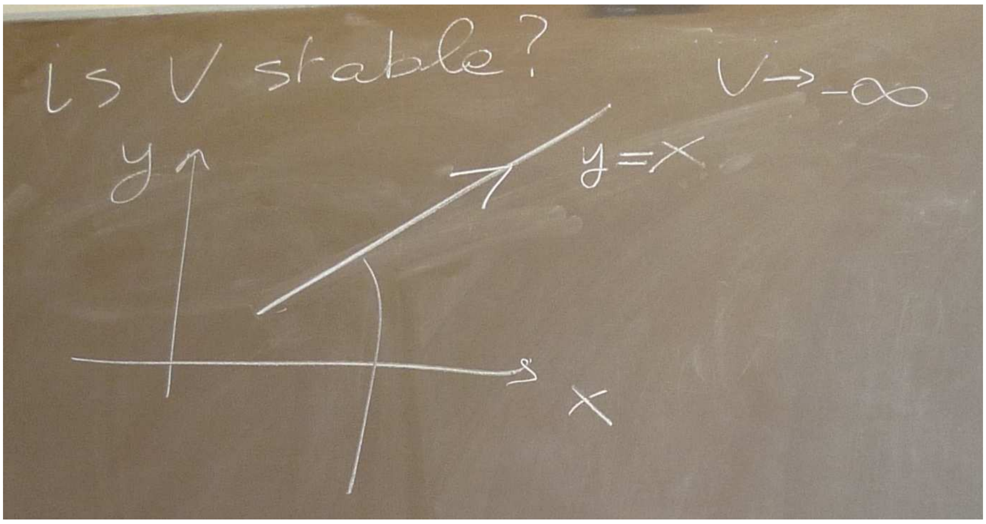

Figure 1. There is a dangerous path that needs to be checked carefully.

$$V \sim \underbrace{\big(2|\mu|^2 + m_{H_u}^2 + m_d^2 - 2b\big)}_{\text{need} > 0} x^2$$

$$b_1 = |\mu|^2 + \frac{m_{H_u}^2 + m_{H_d}^2}{2}, \quad b < b_1.$$

Without soft terms $V = V_D + V_F \geqslant 0$.

Look for min $\frac{\partial V}{\partial x} = \frac{\partial V}{\partial y} = 0$, and look at the Hessian.

$x = y = 0$ is an extremum.

$$H(0,0) = \begin{pmatrix} |\mu|^2 + m_{H_u}^2 & -b \\ -b & |\mu|^2 + m_{H_d}^2 \end{pmatrix}$$

$$\mathrm{tr}(H) = 2b_1 > 0$$

At least one eigenvalue is positive.

det $(H) < 0 \rightarrow x = y = 0$ saddle point, OK. Electroweak symmetry breaking.

$$b^2 > \left(|M|^2 + m_u^2\right)(|M|^2 + m_{H_d}^2) \stackrel{\text{def}}{=\!=} b_2^2$$

Summary.

$$(|\mu|^2 + m_{H_u}^2)(|M|^2 + m_{H_d}^2) < 0$$

$$> 0$$

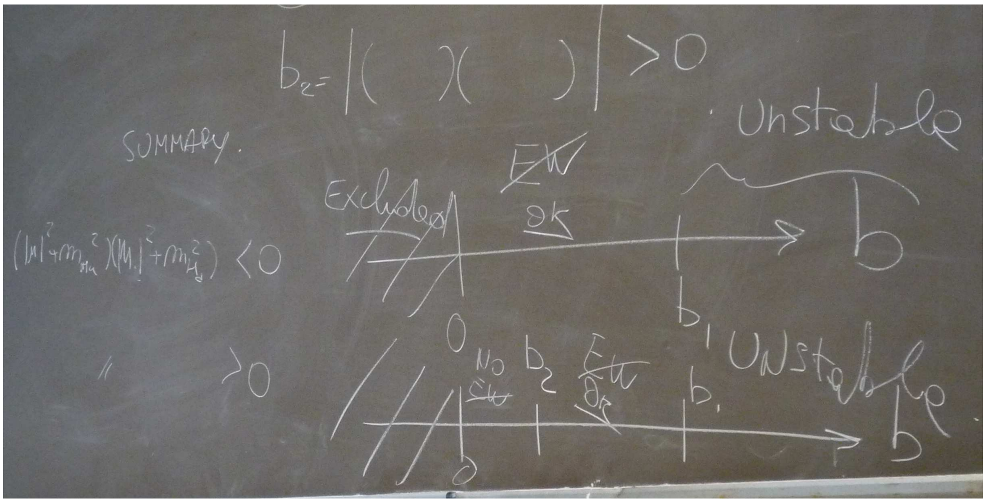

Figure 2.

$$V_{\text{higgs}} = V(x_0 y_0) + m_1^2\, \delta x'^2 + m_2^2 \delta x''^2$$

Higgs masses:

$$\begin{aligned} m_{h^0}^2 &= \frac{1}{2}\left(m_{A^0}^2 + m_{Z^0}^2 - \sqrt{\left(m_{A^0}^2 - m_{Z^0}^2\right)^2 + 4 m_{Z^0}^2 m_A^2 \sin^2(2\beta)} \right) \\ m_{H^0}^2 &= \\ m_{A_0}^2 &= 2\,b / \sin(2\beta) \\ m_{H^\pm}^2 &= \end{aligned}$$

Some signals at the LHC

pp. The LHC is a gluon–gluon collider.

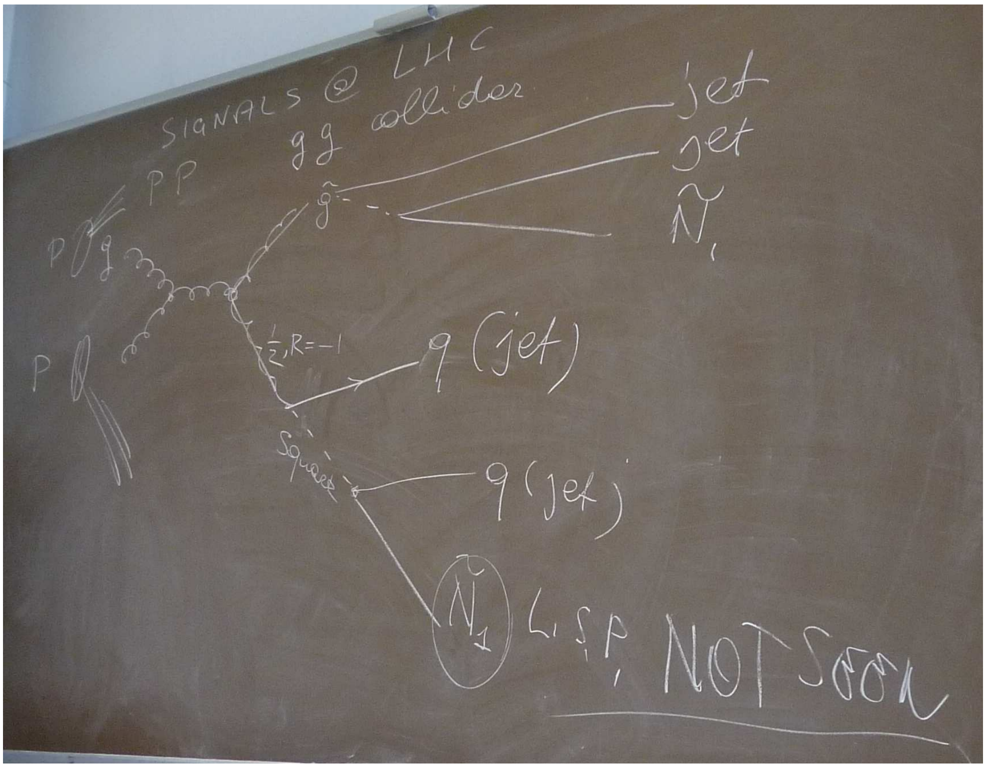

Figure 3.

Expect: 4 jets, missing energy $> 2m_{\tilde{N}_1}$. No leptons.

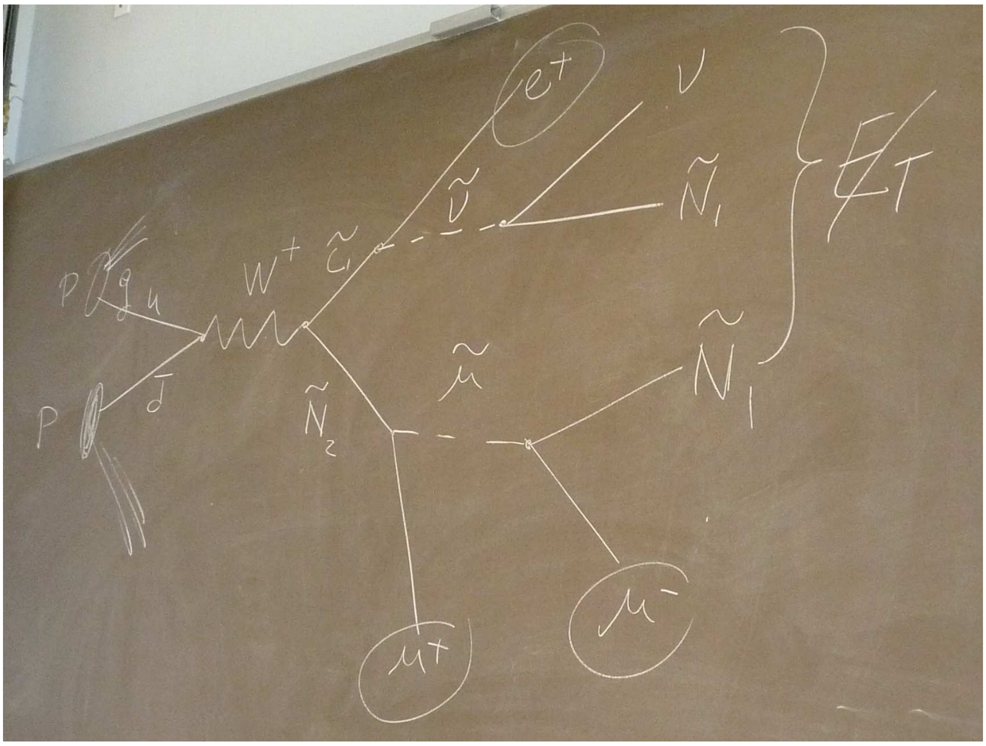

Figure 4. Trileption + missing transverse energy.