2009–05–06

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$$\mathcal{L} = \text{“}\ F^2 + (\mathrm{D}\phi)^2 + \bar{\psi}\,\not{\mathrm{D}}\psi + \bar{\psi}\phi\psi + \phi^2 + \phi^3 + \phi^4\ \text{”}$$

$$\begin{array}{lll} \psi^i, & \phi^i & \text{same mass!} \\ A^a_\mu, & \lambda^a_\alpha & \text{massless} \end{array}$$

$$\mathcal{L}_{\text{gauge}} = -\frac{1}{4}(F^a_{\mu\nu})^2 - \mathrm{i}\,\bar{\lambda}^a\bar{\sigma}^\mu \mathrm{D}_\mu\lambda^a + \underbrace{m\lambda^a_\alpha\lambda^{a\alpha} + \mathrm{cc}}_{\substack{\text{Gauge \& Lorentz} \\ \text{Not susy}}}$$

$$\delta\lambda^a \sim f^{abc}\lambda^b\alpha^c$$

$$\lambda^a\lambda^b = +\lambda^b\lambda^a$$

If $\mathcal{L}$ is gauge invariant, supersymmetry invariant, but has solutions that break the symmetry, then we have spontaneous symmetry breaking.

$$[\delta_{\varepsilon_1}, \delta_{\varepsilon_2}]\phi = \mathrm{i}\underbrace{\left(\varepsilon_i^\dagger\sigma^\mu\varepsilon_2 - \varepsilon_2^\dagger\sigma^\mu\varepsilon_1\right)}_{=\text{some vector } a^\mu}\partial_\mu\phi$$

There is a unitary operator $\hat{\mathcal{P}}^\mu$: $\mathcal{J} \to \mathcal{J}$. $\mathcal{J}$: multiparticle states.

$$\left[a^\mu\hat{\mathcal{P}}_\mu, \phi\right] = \mathrm{i}a^\mu\partial_\mu\phi = \delta_{a^\mu}\phi$$

There is something called the super-charge Q_α, which is an operator satisfying

$$[Q_\alpha, \phi] = \psi_\alpha, \quad \left[Q^\dagger_{\dot{\alpha}}, \phi\right] = 0$$

$(x^\mu, \theta_\alpha, \bar{\theta}^{\dot{\alpha}})$: superspace.

$$\left\{Q_\alpha, Q^\dagger_{\dot{\alpha}}\right\} = \sigma^\mu_{\alpha\dot{\alpha}} \mathcal{P}_\mu$$

$$\mathcal{P}_\mu = \int T_{0\mu} \mathrm{d}^3 \boldsymbol{x}$$

$$\delta\mathcal{L} = \partial_\mu K^\mu$$

The Hamiltonian $H \geqslant 0$. $\mathcal{P}_0 = H$. $\mathcal{P}_0 \sim \left\{Q_\alpha, Q^\dagger_{\dot{\alpha}}\right\} \bar{\sigma}^{0\alpha\dot{\alpha}}$.

Spontaneous supersymmetry breaking

$$\exists_{\alpha=1,2} \quad Q_\alpha |\mathrm{vac}\rangle \neq 0 \quad \Leftrightarrow \quad E_{\mathrm{vac}} \gneqq 0$$

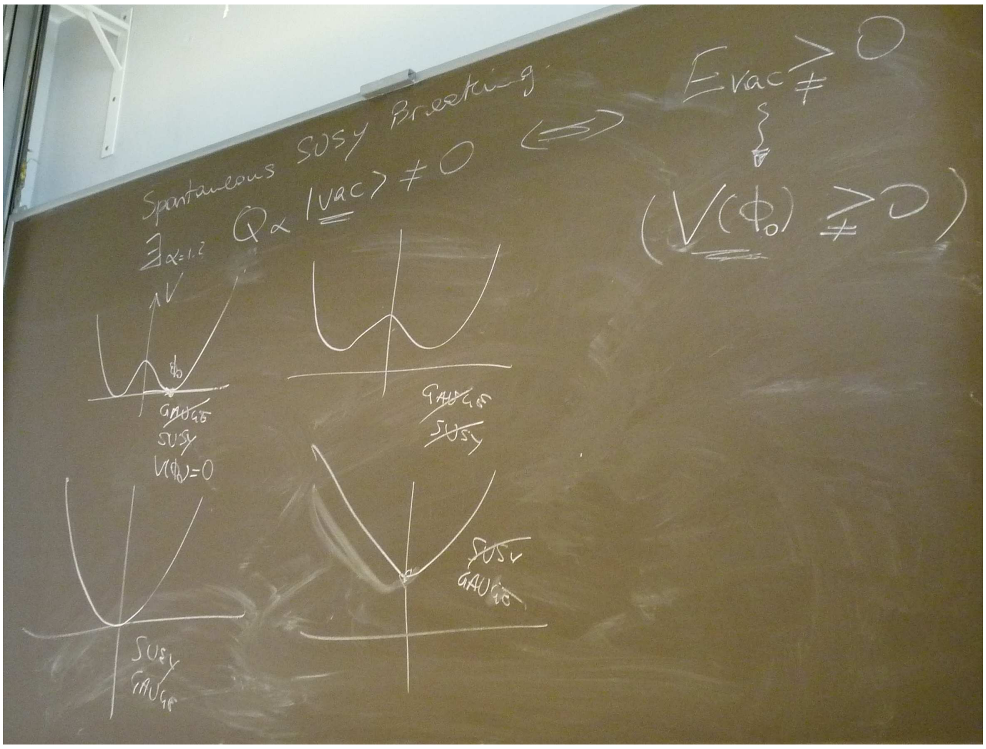

Figure 1.

$\exists ? W(\phi)$ such that

$$V(\phi) = \sum_i \left|\frac{\partial W}{\partial \phi_i}\right|^2 \geqslant 0$$

has a minimum

$$\frac{\partial}{\partial \phi_i} V = 0 \quad \text{such that}\, V(\phi_0) > 0?$$

Only one ϕ, ψ, F, $W(\phi) = a + b\phi + c\phi^2 + d\phi^3$.

$$V = |W'(\phi)|^2$$

$W' = b + 2c\phi + 3d\phi^2 = 0$ unless $b \neq 0, c = d = 0$.

$$V = |b|^2 = \text{const} \neq 0$$

Polony model.

Instead of looking at spontaneous supersymmetry breaking,

$$\mathcal{L}_{\text{MSSM}}(\phi_{\text{MSSM}}) + \mathcal{L}_{\text{extra}}(\phi_{\text{MSSM}}, \phi_{\text{extra}})$$

If the extra fields are very heavy $\gg$ TeV,

$$\mathcal{L}_{\text{effective}} = \mathcal{L}_{\text{MSSM}}(\phi_{\text{MSSM}}) + \mathcal{L}_{\text{soft}}(\phi_{\text{MSSM}}),$$

where $\mathcal{L}_{\text{soft}}$ breaks supersymmetry explicitly.

$$\phi(x) = \int \mathrm{d}k \Big(a_k \mathrm{e}^{\mathrm{i}kx} + a_k^\dagger \mathrm{e}^{-\mathrm{i}k\cdot x} \Big)$$

$$\phi(x+v) = \phi(x) + v^\mu \partial_\mu \phi(x)$$

$$\phi(x+v) = \int \mathrm{d}k \big(a_k\, \mathrm{e}^{\mathrm{i}kx + \mathrm{ikv}} + \cdots \big)$$

$$a_k \to \mathrm{e}^{\mathrm{i}kv} a_k, \quad a_k^\dagger \to \mathrm{e}^{-\mathrm{i}kv} a_k^\dagger$$

$$\prod_{p=0}^{\infty} \left\{ a_{k_1}^\dagger \ldots a_{k_p}^\dagger |0\rangle \right\}$$

$$v^\mu \mathcal{P}_\mu \underbrace{\Big(a_{k_1}^\dagger \ldots a_{k_p}^\dagger |0\rangle \Big)}_{|\psi\rangle} = v^\mu (k_{1\mu} + \cdots + k_{p\mu}) |\psi\rangle$$

$$\mathrm{e}^{\mathrm{i}v\mathcal{P}} |\psi\rangle \to \mathrm{e}^{\mathrm{i}v^\mu (k_1 \ldots k_p)_\mu}$$

$$\mathrm{e}^{\mathrm{i}v\mathcal{P}} \phi(x)\, \mathrm{e}^{-\mathrm{i}v\mathcal{P}} \sim \mathrm{i}\, [v\mathcal{P}, \phi] = \mathrm{i}v^\mu \partial_\mu \phi$$

Simple example WZ

$$W = \frac{m}{2}\phi + \frac{y}{3}\phi^3 \quad (m, y \text{ real}), \quad V = |W'|^2 = m\phi^*\phi + m\,y\left(\phi\phi^{*2} + \phi^2\phi^*\right) + y^2(\phi\phi^*)^2$$

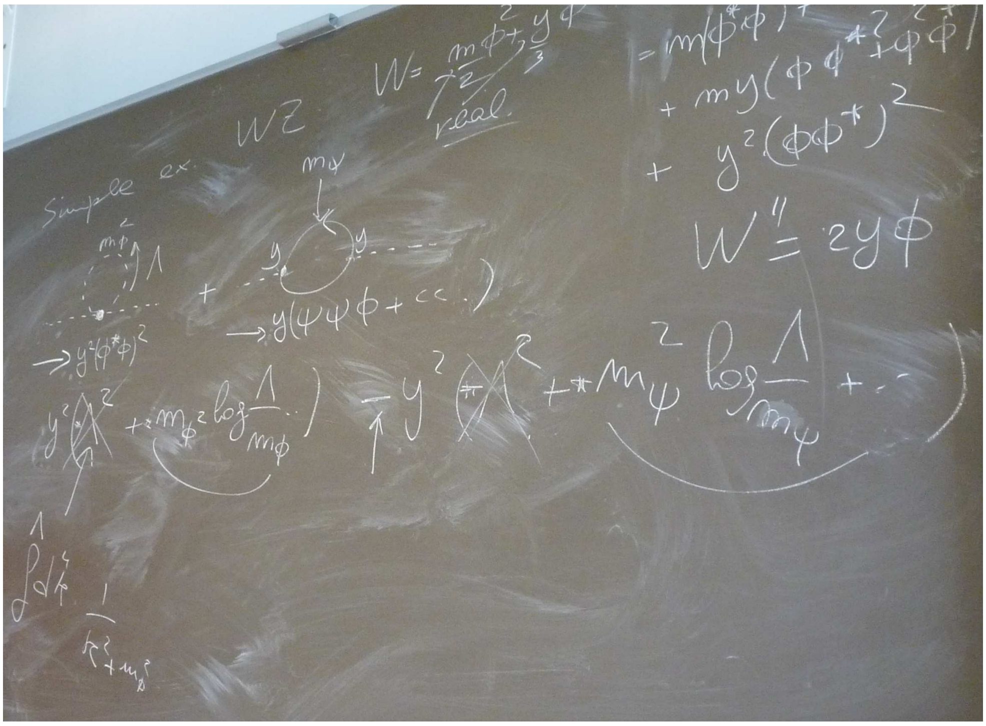

Figure 2.

$\mathcal{L}_{\rm WZ} + \xi\phi\psi\psi + \text{cc}$ would break the cancellation of Λ^2. This term is no good. Not soft.

$$\mathcal{L}_{\rm WZ} + \mu^2\phi^*\phi \text{ soft}$$

A term is soft if it has dimension < 4.

$m\lambda\lambda$. Home there is a better theory at higher energies that generates it.

$m_\phi^2\phi^*\phi + m\lambda\lambda + m'^2\phi^2 + \text{complex conjugate} + \phi^3$

SU(3)	SU(2)	U(1)
8 gluons	$W^\pm, Z, \gamma$	
8 gluinos	$\tilde{W}^\pm, \tilde{Z}, \gamma$	

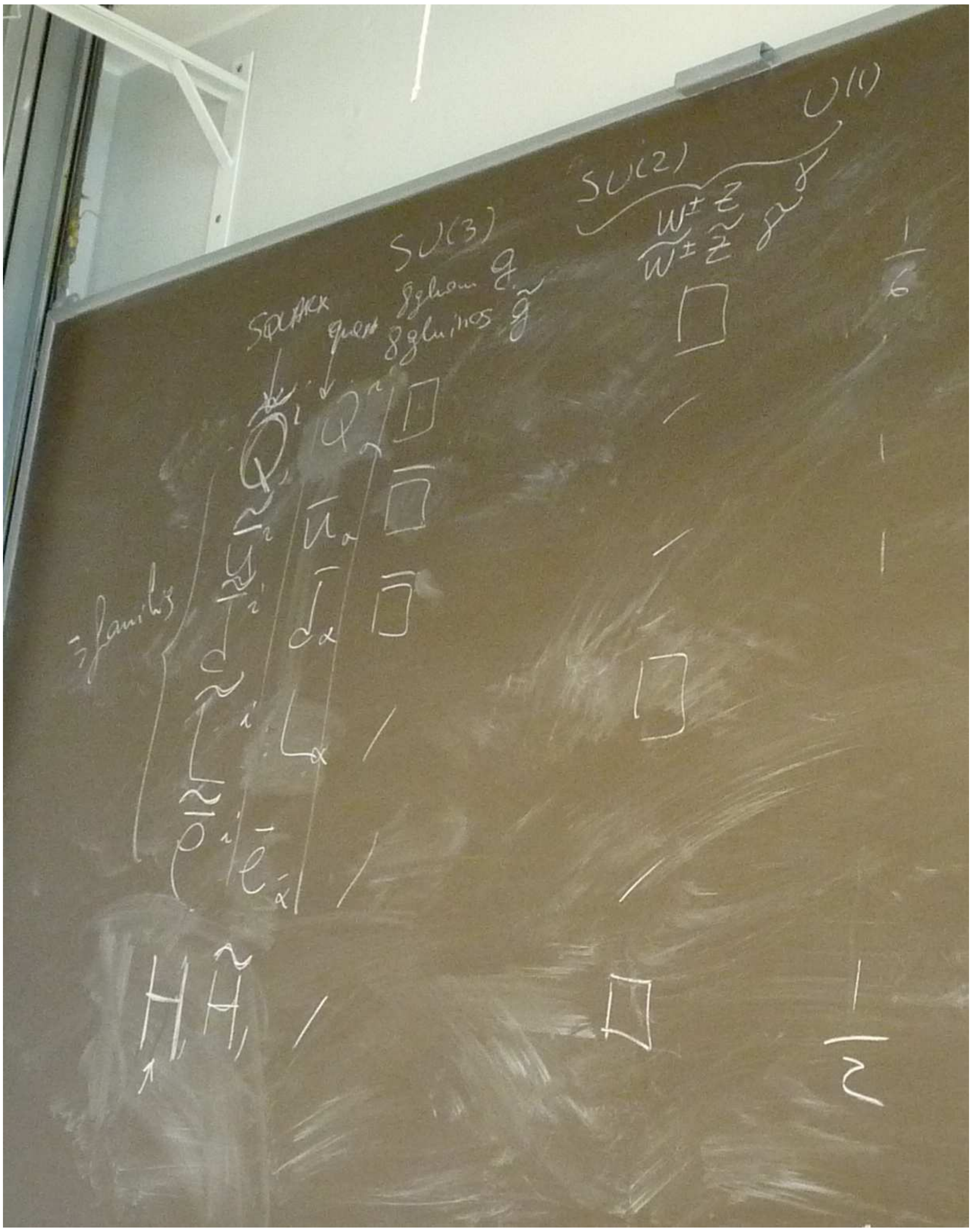

Figure 3.

We will need at least two Higgs fields $(H, \tilde{H})$

$$\mathcal{L} = (\partial\phi)^2 + \bar{\psi}\sigma\partial\phi - \frac{1}{2}\psi\psi\frac{\partial^2 W}{\partial\phi\partial\phi} - \underbrace{\left|\frac{\partial W}{\partial\phi^2}\right|^2}_{V}$$

Pick $W(\phi, \ldots)$. Superpotential $W(\tilde{Q}, \tilde{\bar{u}}, \tilde{\bar{d}}, \tilde{L}, \tilde{\bar{e}}, H\,[\text{how many?}])$.

1) Holomorphic

2) Lorentz invariant (trivial)

3) At most cubic

4) Gauge invariant