2009–05–05

Lecturer: Gabriele Ferretti

May 28 start 14:00 Lindroos, Persson, Pettersson, Neziraj

May 29 start 10:00 Björnsson, Schultz, Kleväng, Lindkvist.

Chiral "super" field $=(\phi^i, \psi_\alpha^i, F^i)$. $\mathbb{C}$, Weyl, auxiliary.

Off-shell:

$$\mathcal{L} = -\partial_\mu \phi^{i*}\, \partial \phi_i - \mathrm{i}\psi^{\dagger i}\bar{\sigma}^{\,\mu}\partial_\mu \psi + F^{*i}F_i - \frac{1}{2}\frac{\partial^2 W}{\partial\phi_i \partial\phi_j}\psi_i\psi_j + \frac{\partial W}{\partial \phi_i}F_i + \text{complex conjugate}$$

$W(\phi^1, \ldots, \phi^n)$ hlomorphic $\frac{\partial}{\partial \phi_i^\dagger}W = 0$. Superpotential.

On shell:

$$\mathcal{L} = -(\partial\phi)^2 - \psi\not{\partial}\psi - \frac{1}{2}\frac{\partial^2 W}{\partial\phi\partial\phi}\psi\psi - \left|\frac{\partial W}{\partial \phi^i}\right|^2$$

Can gauge symmetry and supersymmetry be combined?

G: $[T^a, T^b] = \mathrm{i}\, f^{abc}T^c$, f^{abc} real.

$$\mathcal{L} = -\frac{1}{4}F_{\mu\nu}^a F^{a\,\mu\nu}$$

$$F_{\mu\nu}^a = \partial_\mu A_\nu^a - \partial_\nu A_\mu^a + g\, f^{abc}A_\mu^b A_\nu^c$$

This clearly cannot be supersymmetric, since there is only one gauge boson. For every boson we need a fermion.

Introduce a new field λ_α^a, Weyl fermion. $\alpha = 1, 2$. $a = 1, \ldots, \dim(G)$. Gaugino.

$$\mathcal{L} = -\frac{1}{4}F_{\mu\nu}^a F^{a\,\mu\nu} - \mathrm{i}\lambda^{\dagger a}\bar{\sigma}^{\,\mu}\mathrm{D}_\mu \lambda^a$$

1) $\mathcal{L}$ is already interacting.

2) There are no other terms we can add.

$$[A_\mu] = 1, \quad [\lambda] = \frac{3}{2}$$

$m\lambda^2$ + complex conjugate allowed by gauge invariance and renormalisation, but not supersymmetry.

$$\delta A_\mu^a = \frac{1}{\sqrt{2}}\big(\varepsilon^\dagger \bar{\sigma}_\mu \lambda^a + \text{complex conjugate}\big)$$

$$\delta\lambda_\alpha^a = \ldots A_\mu^a$$

Interlude: A transforms in a rather unpleasant way:

$$\delta_\Lambda A_\mu^a = \partial_\mu \Lambda^a + g\, f^{abc}\, A_\mu^b \Lambda^c$$

$$\delta_\Lambda F_{\mu\nu}^a = g\, f^{abc} F_{\mu\nu}^b \Lambda^c$$

$$\delta_\Lambda \lambda_\alpha^a = g\, f^{abc}\lambda_\alpha^b \Lambda^c$$

|||.

$$\delta\lambda_\alpha^a = (\sigma^\mu\bar{\sigma}^\nu\varepsilon)_\alpha F_{\mu\nu}^a \cdot \frac{\mathrm{i}}{2\sqrt{2}}$$

$$[\delta,\delta] = \text{translation} + \text{equation of motion}$$

on shell.

$$\delta F_{\mu\nu} = 0$$

$$\boldsymbol{E}^a, \boldsymbol{B}^a$$

$$\mathrm{tr}\, \boldsymbol{E}^2 + \boldsymbol{B}^2$$

$$\delta F_{\mu\nu}^a \neq 0 \Rightarrow \text{not observable}$$

$$E_i^a = F_{i0}^a$$

$$\delta(E_i^a E_i^a) = 2\delta E_i^a \cdot E_i^a = 2\, g\, f^{abc} E_i^b \Lambda^c E_i^a = 0$$

The structure constants f^{abc} totally antisymmetric, E_i^b and E_i^a commute.

$\mathrm{D}_\mu F^{\mu\nu} = 0$ — the equation is gauge invariant, even though $D_\mu F^{\mu\nu}$ is not.

$$\delta \mathrm{D} F \propto \mathrm{D} F = 0$$

Off shell

$$A_\mu^a, \lambda^a, D^a$$

D^a is auxiliary field. D^a is real.

$$\mathcal{L}_{\text{off}} = \mathcal{L}_{\text{on}} + D^a D^a$$

Equation of motion: $D^a = 0$.

$$\delta_\Lambda D^a = g\, f^{abc}\, D^b \Lambda^c$$

$$\delta D^a = \frac{\mathrm{i}}{\sqrt{2}} \left(\varepsilon^\dagger \bar{\sigma}^\mu \mathrm{D}_\mu \lambda^a - \mathrm{D}_\mu \lambda^{a\dagger} \bar{\sigma}^\mu \varepsilon\right)$$

$$\delta\lambda_\alpha^a = (\sigma^\mu\bar{\sigma}^\nu\varepsilon)_\alpha F_{\mu\nu}^a \cdot \frac{\mathrm{i}}{2\sqrt{2}} + \frac{1}{\sqrt{2}}\, \varepsilon_a D^a$$

This works in dimensions $D = 3, 4, 6, 10$.

To construct the most general $\mathcal{N} = 1, D = 4$ supersymmetric gauge thery WZ + Gauge

$\phi_i \leftarrow$ think of it as a column vector, $i = 1, \ldots, n$.

$$\delta_\Lambda \phi_i = \mathrm{i} g\, \Lambda^a T_i^{aj} \phi_j$$

$$\mathrm{D}_\mu \phi = (\partial_\mu - \mathrm{i}\, g\, A_\mu)\phi$$

$$|\partial_\mu \phi|^2 \rightarrow |\mathrm{D}_\mu \phi|^2$$

We must “covariantise” the Wess-Zumino model:

$$\mathcal{L} = -\left(\mathrm{D}_\mu\phi\right)^*_i\left(\mathrm{D}_\mu\phi\right)^i - \mathrm{i}\,\psi\,\bar{\sigma}\,\mathrm{D}_\mu\psi + F^\dagger F$$

$$\delta\big(F^\dagger F\big) = \big(\delta F^\dagger\big)\cdot F + F^\dagger\delta F$$

$$\delta F = \mathrm{i}\,g\,T^a\Lambda^a\cdot F$$

$$\delta F^\dagger - \mathrm{i}\,g\,\underset{\substack{\text{row}\\ \text{vector}}}{F^\dagger}\,T^a\Lambda^a$$

$$\delta\big(F^\dagger F\big) = -\,\mathrm{i}\,g\ldots = 0$$

$$\mathcal{L} = -\left(\mathrm{D}_\mu\phi\right)^*_i\left(\mathrm{D}_\mu\phi\right)^i - \mathrm{i}\,\psi\,\bar{\sigma}\,\mathrm{D}_\mu\psi + F^\dagger F - \frac{1}{2}\psi^i\psi^j\,\frac{\partial^2 W}{\partial\phi^i\partial\phi^j} + F\,\frac{\partial W}{\partial\phi} + \text{complex conjugate}$$

will be gauge invariant if and only if W itself is gauge invariant.

$\phi^\dagger\phi$ is not allowed in W.

Albelian $\equiv \mathrm{U}(1)$ gauge field $\phi_1, \ldots, \phi_n$ with charges $q_1, \ldots, q_n$

$$(\partial_\mu + \mathrm{i}q_i\,e\,A_\mu)\phi_i$$

A term $\phi_i\phi_j\phi_k$ will be allowed in W if and only if $q_i + q_j + q_k = 0$. Why?

$$\delta(\phi_i\phi_j\phi_k) = \underbrace{(q_i + q_j + q_k)}_{=0}\phi^3.$$

$\delta_\varepsilon A, \delta_\varepsilon\lambda, \delta_\varepsilon D$ as before, cnnot be changed.

$$\delta_\varepsilon\phi = \varepsilon\psi, \quad \delta_\varepsilon\psi = \mathrm{i}\big(\sigma^\mu\varepsilon^\dagger\big)_\alpha\mathrm{D}_\mu\phi_i + \varepsilon_\alpha F_i, \quad \delta_\varepsilon F = \mathrm{i}\,\varepsilon^\dagger\bar{\sigma}^{\,\mu}\mathrm{D}_\mu\psi_i + \sqrt{2}\,g(T^a\phi)_i\varepsilon^\dagger\lambda^{a\dagger}$$

$$\underset{\substack{\text{Covariantised}\\ \partial_\mu\to\mathrm{D}_\mu\\ W\,\text{is gauge}\\ \text{invariant}}}{\mathcal{L}_{\mathrm{WZ}}} + \mathcal{L}_{\text{gague}} + \mathcal{L}'$$

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$$\phi^\dagger T^a\psi\lambda^a$$

$$\lambda^\dagger\psi^\dagger T^a\phi$$

$$D^a\phi^\dagger T^a\phi$$

Gauge, Lorentz, $D = 4$.