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$$\psi = \begin{pmatrix} \xi_\alpha \\ \chi^{\dagger\dot{\alpha}} \end{pmatrix}, \quad \bar{\psi} = \begin{pmatrix} \chi^\alpha & \xi^\dagger_{\dot{\alpha}} \end{pmatrix}$$

Dirac mass term:

$$m\,\bar{\psi}\,\psi = m\Big(\chi^\alpha\xi_\alpha + \xi^\dagger_{\dot{\alpha}}\chi^{\dagger\dot{\alpha}}\Big) = m\big(\chi\xi + \xi^\dagger\chi^\dagger\big)$$

$$\chi_\alpha\xi^\alpha = -\,\chi^\alpha\,\xi_\alpha$$

$$\chi\xi = +\,\xi\chi$$

$$\xi\chi = \xi^\alpha\chi_\alpha = \xi^1\chi_1 + \xi^2\chi_2 = \xi_2\chi_1 - \xi_1\chi_2 = -\,\chi_1\xi_2 + \chi_2\xi_1 = \chi^2\xi_2 + \chi^1\xi_1 = \chi\xi$$

$$\xi\,\xi = \xi^2 \neq 0$$

$$\xi\xi = -\,2\xi_1\xi_2 = 2\,\xi_2\xi_1$$

Majorana

$$\psi_M = \begin{pmatrix} \xi_\alpha \\ \xi^{\dagger\dot{\alpha}} \end{pmatrix}, \quad m\,\bar{\psi}_M\psi_M = m\big(\xi\xi + \xi^\dagger\xi^\dagger\big)$$

Majorana mass.

$$\bar{e}_R\,e_R = 0, \quad e_R = \begin{pmatrix} \xi_\alpha \\ 0 \end{pmatrix}$$

$$\xi \to \mathrm{e}^{\mathrm{i}\alpha}\xi, \quad \chi \to \mathrm{e}^{-\mathrm{i}\alpha}\chi \quad \Rightarrow \quad \psi_D \to \mathrm{e}^{\mathrm{i}\alpha}\psi_D$$

Majorana masses are allowed only for particles transforming under a *real representation* of the gauge group. The *gluino* has a mass term of this type.

Thye simples Susy Lagrangian in foru dimension is the Wess-Zumino: Complex scalar ϕ and a Weyl spinor ψ_α. The free Lagrangian:

$$\mathcal{L}_F = -\,\partial_\mu\phi^*\,\partial^\mu\phi - \mathrm{i}\psi^\dagger_{\dot{\alpha}}\bar{\sigma}^{\,\mu\dot{\alpha}\alpha}\partial_\mu\psi_\alpha$$

There is a $\mathrm{U}(1)^2$ global symmetry.

Symmetry:

$$\delta\mathcal{L} = \partial_\mu K^\mu \quad \Rightarrow \quad \exists\,\partial_\mu J^\mu = 0, \;\; Q = \int \mathrm{d}^3x\, J^0$$

$$\delta\phi = \varepsilon^\alpha\psi_\alpha \quad \varepsilon^1 \text{ and } \varepsilon^2 \text{ are two constant, anticommuting (Grassman) numbers}$$

For any function f

$$f(\varepsilon) = f_0 + f_1\varepsilon_1 + f_2\,\varepsilon_2 + f_3\,\varepsilon_1\varepsilon_2; \quad \varepsilon_1\varepsilon_1 = -\,\varepsilon_1\varepsilon_1 = 0$$

This $\delta\phi$ is a guess.

$\mathcal{N} = 1$ supersymmetry: only one ε^α.

$$\delta\psi_\alpha \stackrel{?}{=\!=} \text{something with } \phi$$

With $\hbar = c = 1, [\phi] = M, [\psi] = M^{3/2} \Rightarrow [\varepsilon] = M^{-1/2}$.

$$\delta\psi^{\alpha} = -\,\mathrm{i}\,\varepsilon^{\dagger}_{\dot{\alpha}}\,\sigma^{\mu\dot{\alpha}\alpha}\,\partial_{\mu}\phi$$

$$\delta\psi_{\alpha} = \mathrm{i}\varepsilon^{\dagger\dot{\alpha}}\sigma^{\mu}_{\alpha\dot{\alpha}}\,\partial_{\mu}\phi = \mathrm{i}\sigma^{\mu}_{\alpha\dot{\alpha}}\varepsilon^{\dagger\dot{\alpha}}\,\partial_{\mu}\phi \equiv \mathrm{i}\big(\,\sigma^{\mu}\varepsilon^{\dagger}\big)_{\alpha}\,\partial_{\mu}\phi$$

$$\delta\mathcal{L} = \partial_{\mu}K^{\mu} \text{ for some } K^{\mu}$$

This leaves the action invariant, $\delta S = \int \mathrm{d}^4x\,\delta\mathcal{L} = 0$. The explicit form of K^{μ} matters when we look for conserved currents. If we are not interested in J_{μ} (and K^{μ}), we can drop all total derivatives.

$$\delta(-\partial^{\mu}\phi\partial_{\mu}\phi^{*}) = -\,\varepsilon\partial^{\mu}\psi\,\partial_{\mu}\phi^{*}0 + \text{complex conjugate}$$

$$\delta\big(-\mathrm{i}\psi^{\dagger}\bar{\sigma}^{\nu}\partial_{\nu}\psi\big) = -\,\varepsilon\sigma^{\mu}\bar{\sigma}^{\nu}\partial_{\nu}\psi\partial_{\mu}\phi^{*} + \psi^{\dagger}\bar{\sigma}^{\nu}\sigma^{\mu}\varepsilon^{\dagger}\partial_{\mu}\partial_{\nu}\phi$$

Check that it chooses

$$[\delta_{\varepsilon_1}, \delta_{\varepsilon_2}]^{\phi}_{\psi} \text{ is a symmetry.}$$

$g = \mathrm{e}^{\mathrm{i}A}$ for group element g and Lie algebra element A. Think of g as metrices. $g_1g_2g_3\Phi$. We want associativity. Infinitesimally, this means that the commutator of the symmetry transformation is a symmetry itself.

$$\delta_{\varepsilon_1}\delta_{\varepsilon_2}\phi = \delta_{\varepsilon_1}(\varepsilon_2\psi) = \varepsilon_2\,\delta_{\varepsilon_1}\psi = \mathrm{i}\,\varepsilon_2\,\sigma^{\mu}\varepsilon_1^{\dagger}\partial_{\mu}\phi$$

$$[\delta_{\varepsilon_1}, \delta_{\varepsilon_2}]\phi = \mathrm{i}\underbrace{\Big(\varepsilon_2\sigma^{\mu}\varepsilon_1^{\dagger} - \varepsilon_1\sigma^{\mu}\varepsilon_2^{\dagger}\Big)}_{a^{\mu}}\partial_{\mu}\phi$$

It's a translation.

$$[\delta_{\varepsilon_1}, \delta_{\varepsilon_2}]\psi_{\alpha} = \mathrm{i}\Big(\sigma^{\mu}\varepsilon_1^{\dagger}\Big)_{\alpha}\varepsilon_2\partial_{\mu}\psi - \mathrm{i}\Big(\sigma^{\mu}\varepsilon_2^{\dagger}\Big)_{\alpha}\varepsilon_1\partial_{\mu}\psi \overset{?}{=\!=} b^{\mu}\partial_{\mu}\psi_{\alpha}\text{: no, } \alpha \text{ at the wrong place.}$$

Not a translation.

$$[\delta_{\varepsilon_1}, \delta_{\varepsilon_2}]\psi_{\alpha} = \mathrm{i}\Big(\varepsilon_1\sigma^{\mu}\varepsilon_2^{\dagger} - \varepsilon_2\sigma^{\mu}\varepsilon_1\Big)\partial_{\mu}\psi_{\alpha} - \Big(\varepsilon_{1\alpha}\varepsilon^{\dagger}_{2\dot{\beta}} - \varepsilon_{2\alpha}\varepsilon^{\dagger}_{1\dot{\beta}}\Big)\underbrace{\bar{\sigma}^{\mu\dot{\beta}\beta}\partial_{\mu}\psi_{\beta}}_{=0 \text{ by eq. o. mot.}}$$

The symmetry closes "on shell" (i.e., using the equation of motion, most often the equation for the fermions).

δ as formulated now acts on ϕ, ψ

- $\delta\mathcal{L} = \partial_{\mu}K^{\mu}$: OK
- $[\delta_1, \delta_2]$ symmetry (translation!) on shell.

Question? Is it possible to introduce extra auxiliary fields F such that δ acts on ϕ, ψ and F, so that it works off-shell too?