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Project: Thursday 28 May 9–17 and Friday 29 May 9–12. Password for course evaluation: "`susyQ`".

Today we start with supersymmetry.

But first Higgs. We have an experimental limit: $m_h > 114\,\mathrm{GeV}$. At Lep, an e^+e^- collider

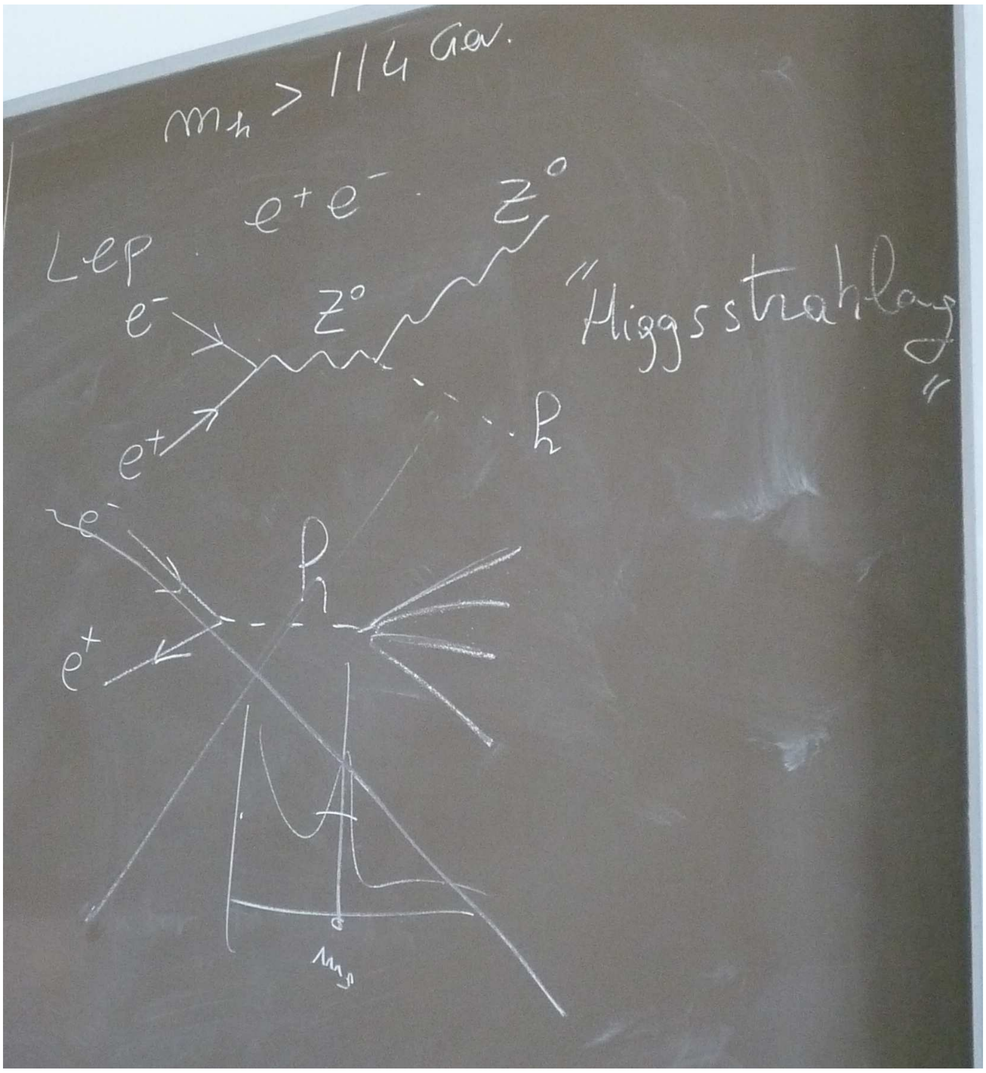

Figure 1. Higgsstrahlung. The lower process does not occur.

Vacuum expectation value v.

$$m \propto v \rightarrow v + h$$

$$m \bar{\psi} \psi \rightarrow m \bar{\psi} \psi + \frac{m}{v} \bar{\psi} \psi h$$

This makes the $e^- e^+ h$ vertex vanishingly small.

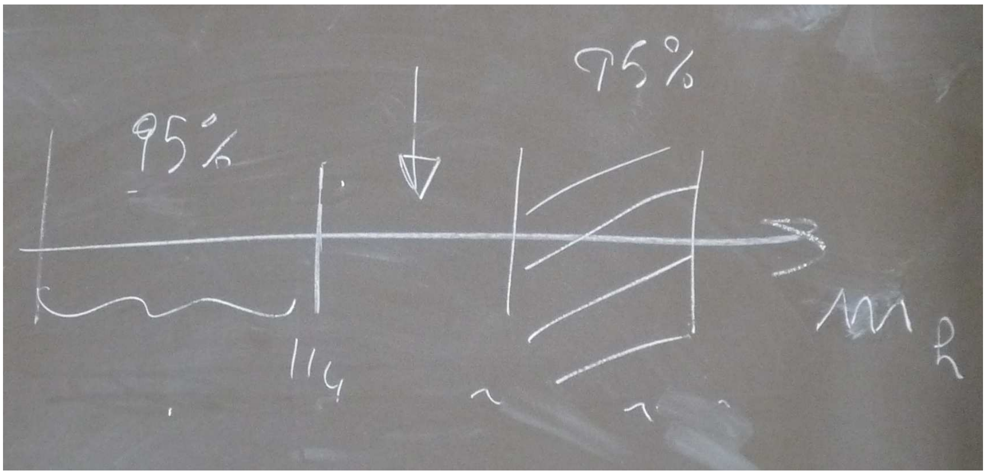

Figure 2. Lep excludes masses lower than 114 GeV with 95% confidence level. The Tevatron takes a higher

The first process you expect is Higgsstrahlung. The second is vector boson fusion.

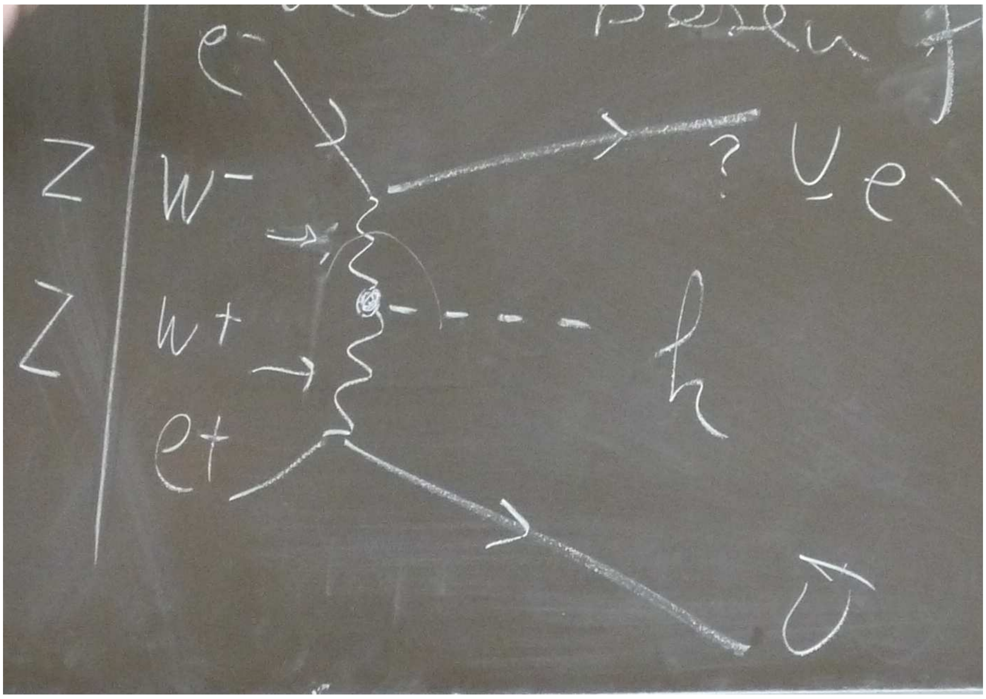

Figure 3. Two reactions in one graph. Either the $W^\pm$ and the $\nu\bar{\nu}$, or the Z and e^-e^+.

Why can we predict the masses of $Z, W^\pm$, but not the Higgs?

$$V = -\mu^2\phi^\dagger\phi + \lambda\left(\phi^\dagger\phi\right)^2, \quad \phi = \frac{1}{\sqrt{2}}\begin{pmatrix} 0 \\ v+h \end{pmatrix}$$

$$V = -\frac{\mu^2}{2}(v+h)^2 + \frac{\lambda}{4}(v+h)^4 = \cancel{c_0} + \underbrace{c_1 h}_{\substack{=0 \\ (\mathrm{min})}} + c_2 h^2 + c_3 h^3 + \frac{\lambda}{4}h^4$$

$$v = \sqrt{\frac{\mu^2}{\lambda}}$$

$$c_1 = -\frac{\mu^2}{2}\cdot 2v\,h + \frac{\lambda}{4}4v^2 h = 0 \quad \Rightarrow \quad v = \sqrt{\frac{\mu^2}{\lambda}}$$

$$c_2 \overset{v}{=\!=} + \mu^2 = \frac{1}{2}m_h^2$$

Nobody knows what the value of λ is. Every other mass is proportional to v, but $v = \sqrt{\mu^2/\lambda}$.

Hadron collider

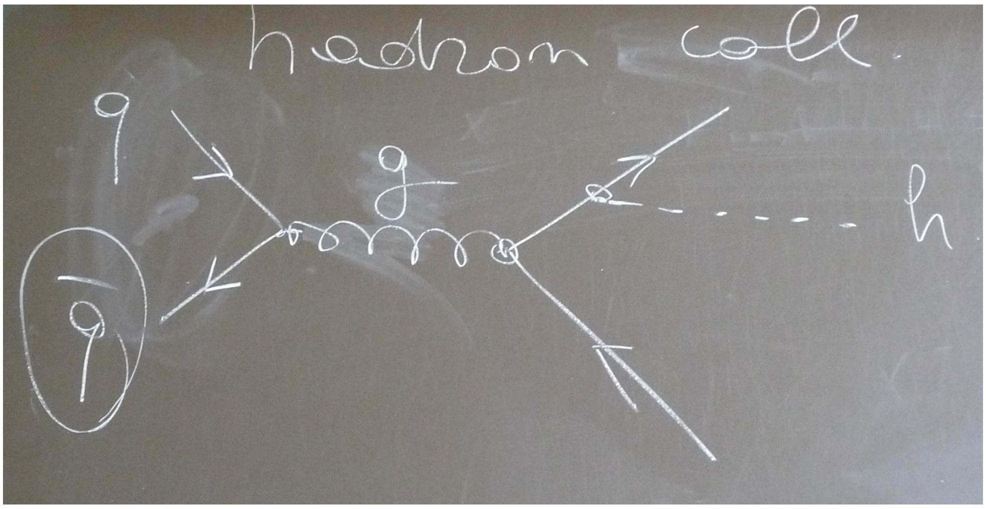

Figure 4. Higgsstrahlung from a top.

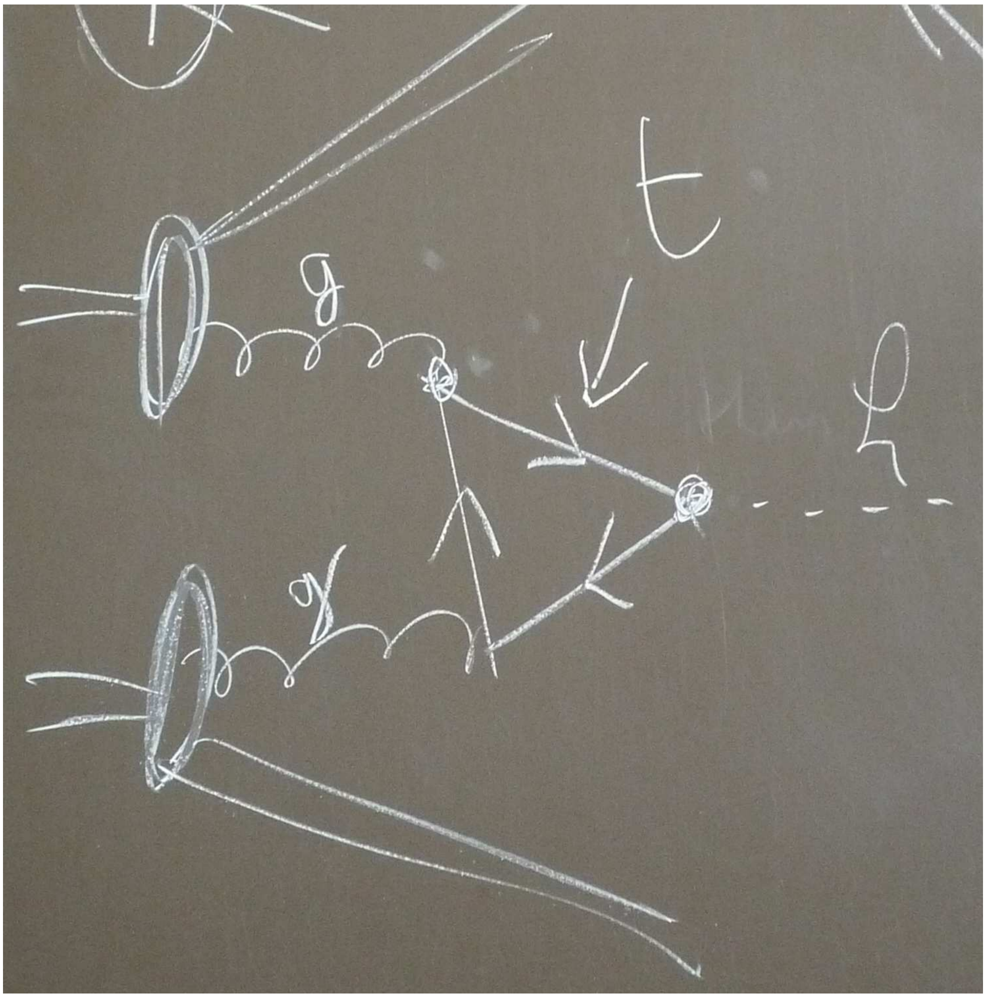

Figure 5. Contribution from a loop.

This is for the production. To actually see the Higgs, we need to see it decay. The decay depends on the mass.

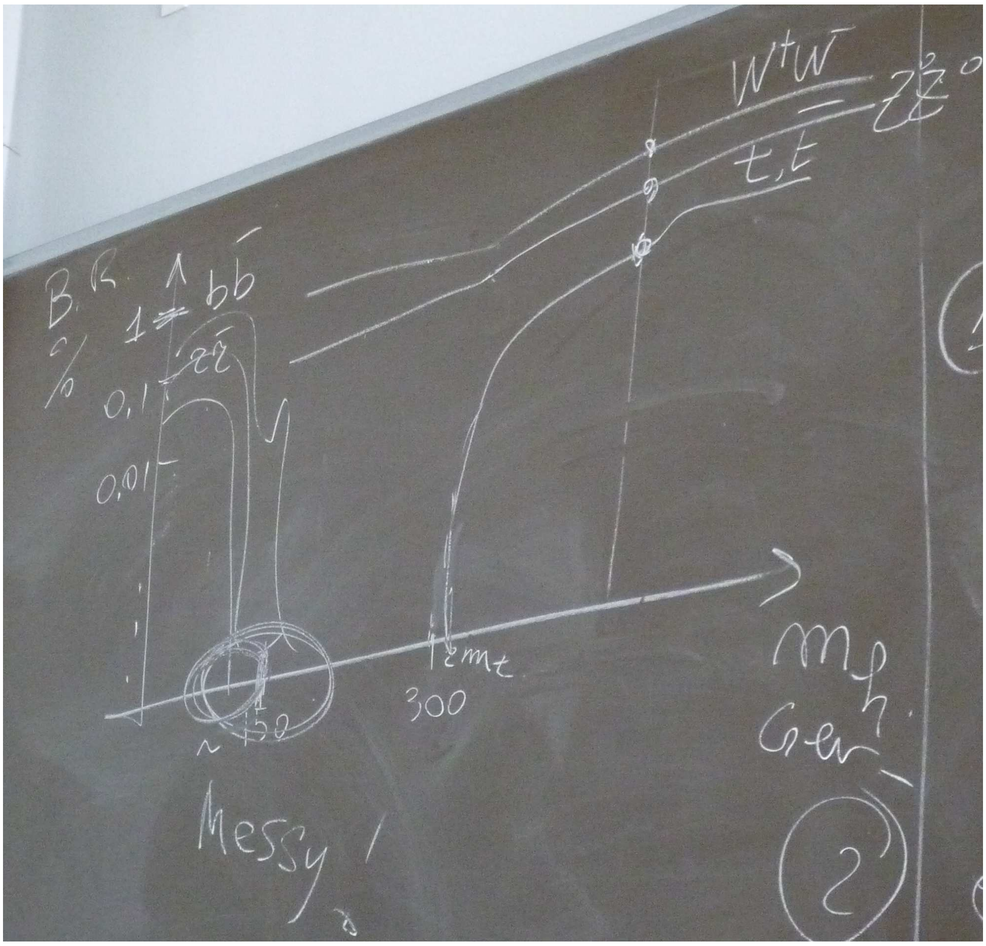

Figure 6.

Supersymmetry

Naturalness problem: In the standard model the Higgs mass gets renormalised.

$$\frac{\mathrm{i}}{p^2 - m_h^2}$$

If this is the propagator of the Higgs, the Higgs mass is the pole of the propagator.

$$(\partial_\mu h)^2 - m_h^2 h^2 + \frac{\lambda}{4} h^4$$

$$\left(p^2 - m_\eta^2\right)\tilde{h} = 1$$

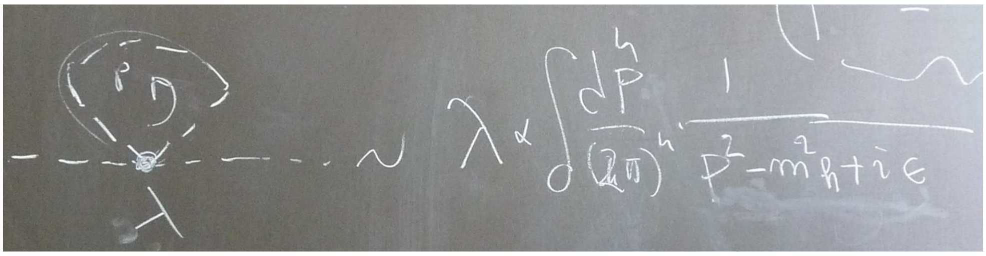

Figure 7.

$$\langle\text{fig7}\rangle \sim \lambda \int\limits_{0<|p|<\Lambda} \frac{\mathrm{d}^4 p}{(2\pi)^4} \cdot \frac{1}{p^2 - m_h^2 + \mathrm{i}\varepsilon} \sim \lambda\Lambda^2$$

Cut-off Λ.

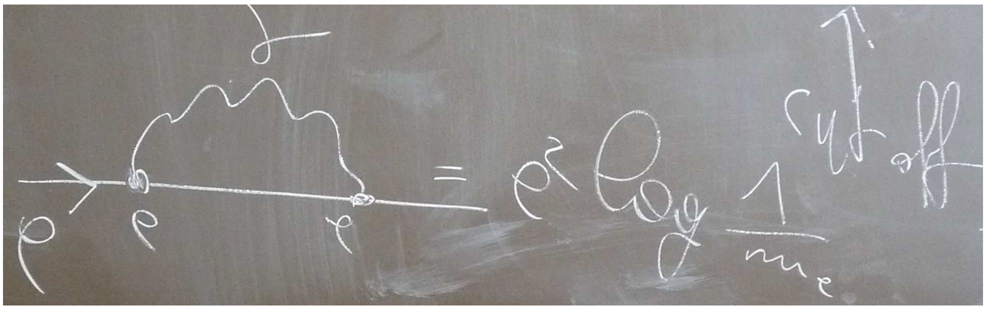

Figure 8. This diagram is divergent like a log. A much milder divergence.

In supersymmetry, it was realised, there are no quadratic divergences.

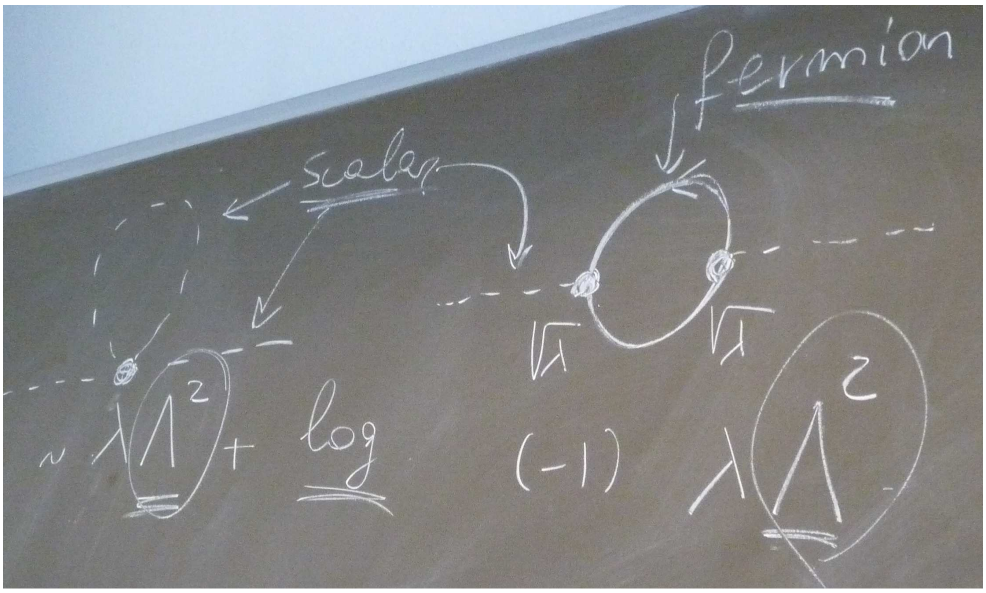

Figure 9. Quadratic divergences are handled in supersymmetry by a fermion loop and a scalar loop having opposite contributions.

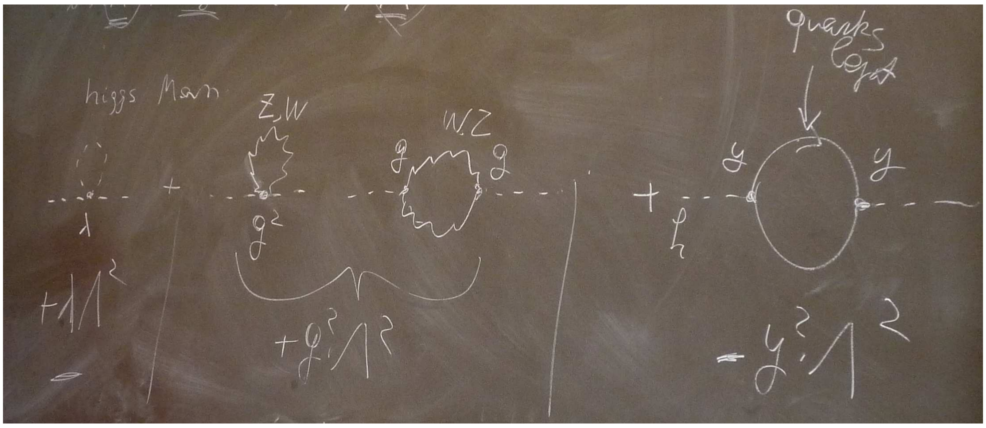

Figure 10. Each sector will have its own sypersymmetric partner: The Higgs has the higgsino, the gluons have gluinos, ...

Supersymmetry: 3 reasons:

1) m_h light.

2) Helps in grand unified theory:

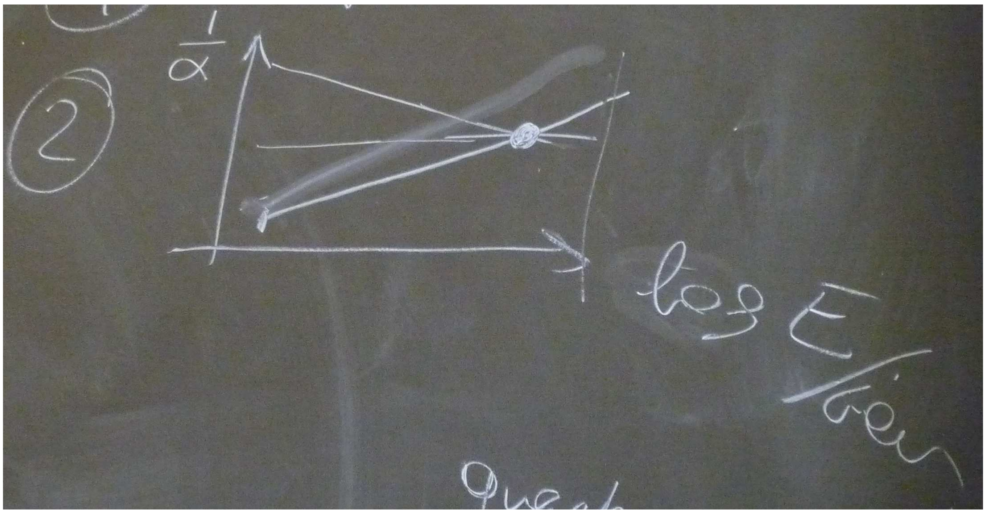

Figure 11. $1/\alpha$ versus E.

Supersymmetry makes the forces unify at one point.

3) Dark matter.

You have to introduce R-parity. A multiplicative quantum number $= +1$ on the matter, and for the superpartners -1. If R-parity is conserved: a) superpartners can only be produced in pairs, and b) the lightest one will be stable (can of course annihilate).

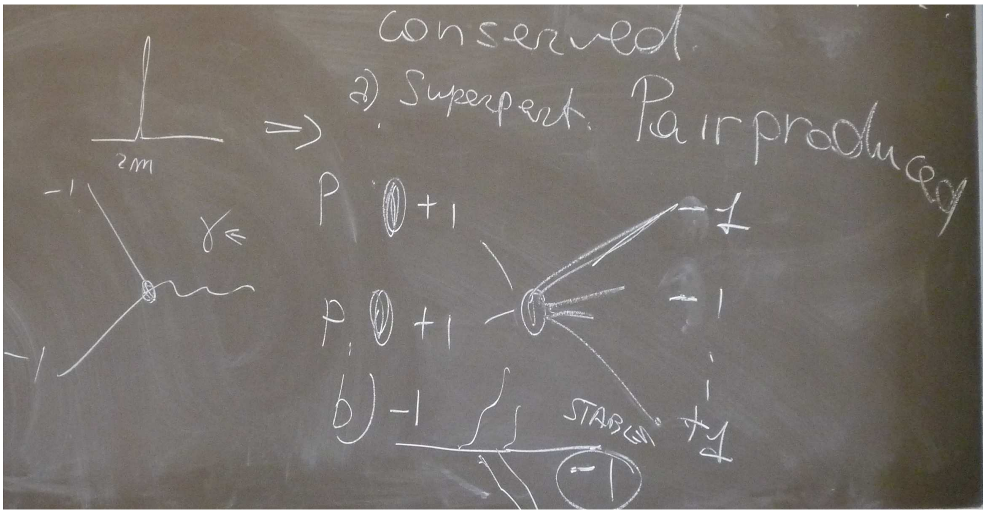

Figure 12.

Notation

$$\gamma^\mu = \begin{pmatrix} 0 & 0 & & \sigma_\mu \\ 0 & 0 & & \\ & & 0 & 0 \\ \bar{\sigma}_\mu & & 0 & 0 \end{pmatrix},$$

$$\sigma_0 = \bar{\sigma}_0 = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}, \quad \sigma_1 = -\bar{\sigma}_1 = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}, \quad \sigma_2 = -\bar{\sigma}_2 = \begin{pmatrix} 0 & -\mathrm{i} \\ \mathrm{i} & 0 \end{pmatrix}, \quad \sigma_3 = \bar{\sigma}_3 = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$$

γ^μ is written in the Weyl matrix.

$$\mathrm{i}\,\gamma^0 \gamma^1 \gamma^2 \gamma^3 = \gamma_5 = \begin{pmatrix} -1 & & & \\ & -1 & & \\ & & 1 & \\ & & & 1 \end{pmatrix}$$

$$P_{L,R} = \frac{1}{2}(1 \pm \gamma_5)$$

$$P_L = \begin{pmatrix} 1 & & & \\ & 1 & & \\ & & 0 & \\ & & & 0 \end{pmatrix}, \quad P_R = \begin{pmatrix} 0 & & & \\ & 0 & & \\ & & 1 & \\ & & & 1 \end{pmatrix}$$

Dirac spinor

$$\Psi = \begin{pmatrix} \xi \\ \chi^\dagger \end{pmatrix}$$

$\xi, \chi^\dagger$: two component Weyl spinor.

$$P_L \Psi = \begin{pmatrix} \xi \\ 0 \end{pmatrix} = \xi \quad \text{which is an abuse of notation...}$$

$$P_R \Psi = \begin{pmatrix} 0 \\ \chi^\dagger \end{pmatrix} = \chi^\dagger$$

ξ_α with $\alpha = 1, 2$ and $\chi^{\dagger\dot{\alpha}}$ with $\dot{\alpha} = \dot{1}, \dot{2}$.

$$\frac{\mathrm{i}}{4}[\gamma^\mu, \gamma^\nu] = J^{\mu\nu} = 4 \times 4 \text{ matrix}$$

$$\left[L^i, L^j\right] = \mathrm{i}\, \varepsilon^{ijk} L^k$$

$$M^{ij} = \varepsilon^{ijk} L^k = -M^{ji}$$

$$\rightarrow \quad [M^{ij}, M^{lm}] = (\delta\delta - \delta\delta) M$$

$$[J^{\mu\nu}, J^{\rho\tau}] = (\eta\eta\ldots) J$$

$$\{\gamma^\mu, \gamma^\nu\} = 2\eta^{\mu\nu}$$

$$J^{\mu\nu} = \begin{pmatrix} \sigma^{[\mu} \bar{\sigma}^{\nu]} & 0 \\ 0 & \bar{\sigma}^{[\mu} \sigma^{\nu]} \end{pmatrix}$$

$$\left[J^{\mu\nu}, \gamma^5\right] = 0$$

$$(\xi^\alpha)^* = \xi^{\dagger\dot{\alpha}}$$

$$\bar{\psi} = \psi^\dagger \gamma^0 = \left(\xi^\dagger_{\dot{\alpha}}, \xi^\alpha\right) \begin{pmatrix} 0 & \mathbf{1} \\ \mathbf{1} & 0 \end{pmatrix} = \left(\xi^\alpha, \ \xi^\dagger_{\dot{\alpha}} \right)$$

Dirac mass

$$m\bar{\psi}\psi = m\left(\chi^\alpha, \xi^\dagger_{\dot{\alpha}} \right) \begin{pmatrix} \xi_\alpha \\ \chi^{\dagger\dot{\alpha}} \end{pmatrix} = m\left(\chi^\alpha \xi_\alpha + \xi^\dagger_{\dot{\alpha}} \chi^{\dagger\dot{\alpha}} \right)$$

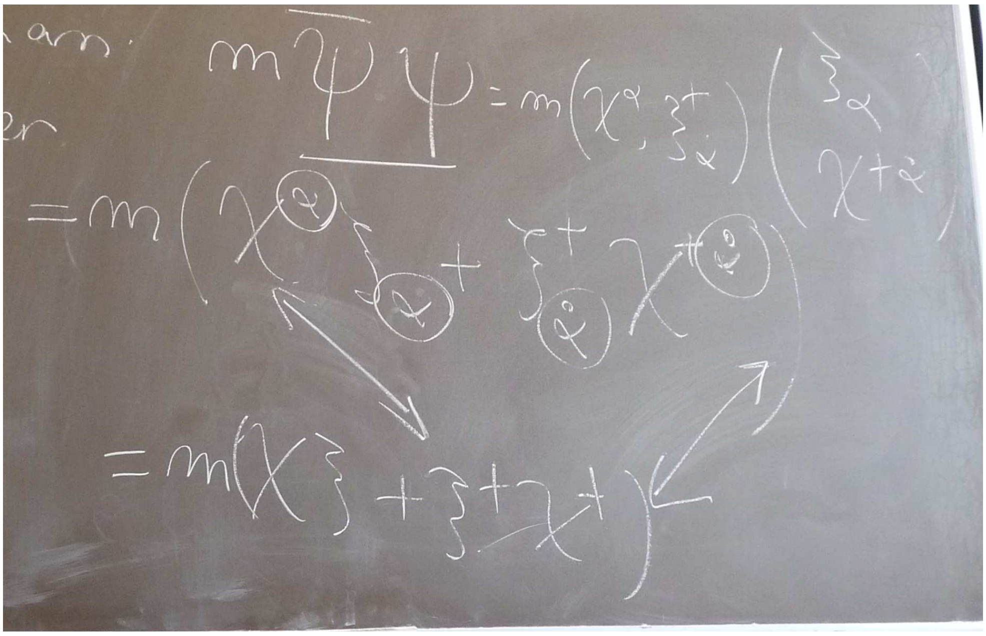

Figure 13.

Wess–Bagger is the bible of supersymmetry notation.

$$= m\big(\chi\xi + \xi^\dagger\chi^\dagger\big)$$

How to raise and lower indices? V^μ, W^μ: $\eta_{\mu\nu}V^\mu W^\nu$. Short hand: $W_\mu = \eta_{\mu\nu}W^\nu$, so $V \cdot W = V^\mu W_\mu$. I am allowed to do this because there is an object η that is invariant under Lorentz transformations. The existence of an invariant tensor is required to make a scalar of two vectors.

Is there an invariant tensor for the Weyl representation? Yes.

You usually think of a Lorentz transformation as a matrix $\Lambda^\mu{}_\nu$. This is not the Lorentz transformation. It is the rule for transforming vectors.

$$T_{\mu\nu} \to \Lambda^\rho{}_\mu \Lambda^\lambda{}_\nu T_{\rho\lambda}$$

Define a multi-index $I = 1, \ldots, 16$ corresponding to combinations of (μ, ν) from $(0,0)$ to $(3,3)$.

$$T_I = \hat{\Lambda}^J{}_I T_J$$

$$\hat{\Lambda}^4{}_7 = \Lambda\,\Lambda$$

$$\phi \to \Lambda_{\text{scalar}}\phi, \quad \Lambda_{\text{scalar}} = 1$$

$$\xi_\alpha \to \Lambda^\alpha_{\frac{1}{2}\beta}\xi_\beta, \quad \chi^\dagger \to \bar{\Lambda}_{\frac{1}{2}}\chi^\dagger$$

$$\exists? \quad \Lambda^\alpha_{\frac{1}{2}\gamma}\Lambda^\beta_{\frac{1}{2}\delta}\varepsilon_{\alpha\beta} = \varepsilon_{\gamma\delta}$$

$$\varepsilon_{12} = \varepsilon_{21} = +1$$

$$\Lambda^\alpha_{\frac{1}{2}\gamma}\Lambda^\beta_{\frac{1}{2}\delta}\varepsilon^{\gamma\delta} = \varepsilon^{\alpha\beta}$$

$$V_\mu = \eta_{\mu\nu}V^\nu$$

$$\xi^\alpha = \varepsilon^{\alpha\beta}\xi_\beta$$

$$\varepsilon_{\alpha\beta}\xi^\alpha\chi^\beta = \xi^\alpha\chi_\alpha$$

$$\chi_\alpha \overset{\text{def}}{=\!=} \varepsilon_{\alpha\beta}\chi^\beta$$