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$$\text{“ } F^2 + \bar{\psi}\,\not{D}\psi + (\mathrm{D}\phi)^2 + \bar{\psi}\phi\psi + \phi^2 + \phi^4 \text{ ”}$$

$$|\mathrm{D}_\mu \phi|^2 \qquad \phi = \phi_0 = \frac{1}{\sqrt{2}}\begin{pmatrix} 0 \\ v \end{pmatrix} \quad \text{masses } W^{\pm}, Z$$

$$\begin{pmatrix} Z_\mu \\ A_\mu \end{pmatrix} = \begin{pmatrix} \cos\theta_{\mathrm{W}} & \ldots \\ \ldots & \ldots \end{pmatrix}\begin{pmatrix} A^3_\mu \\ B_\mu \end{pmatrix}$$

$$W^{\pm} = \frac{A^1 \pm \mathrm{i}\, A^2}{\sqrt{2}}$$

$$\mathrm{D}_\mu = \partial_\mu - \mathrm{i}\,\frac{g}{\sqrt{2}}\big(W^+_\mu\,\tau^+ + W^- \tau^-\big) - \mathrm{i}\,\frac{g}{\cos\theta_W}\,Z^0_\mu\big(\tau^3 - \sin^2\theta_W\,Q\big) - \mathrm{i}\,e\,A_\mu Q + \mathrm{i}\,g_s\,G^a_\mu T^a$$

$$\tau^{\pm} = \frac{1}{\sqrt{2}}\big(\tau^1 \pm \mathrm{i}\,\tau^2\big), \quad Q = \tau^3 + y$$

$$\mathrm{D}_\mu L_{\mathrm{L}}$$

$$\mathrm{D}_\mu e_{\mathrm{R}} = \left(\partial_\mu + \mathrm{i}\,\frac{g\sin^2\theta_{\mathrm{W}}}{\cos\theta_{\mathrm{W}}}\,Z_\mu - \mathrm{i}\,e_\mu\right)e_R$$

$$\text{“ } \bar{\psi}\phi\psi \text{ ”} = -\,\lambda^{ij}_{(d)}\,\bar{Q}^{\,i}_L\phi\,d^i_R - \lambda^{ij}_{(u)}\,\bar{Q}^{\,i}\phi^\dagger u^j - \lambda^{ij}_{(e)}\,\bar{L}^{\,i}\phi\,e^j_R + \text{“}\nu\text{”?}$$

$\mathrm{SU}(3) \times \mathrm{SU}(2) \times \mathrm{U}(1)$

$$\overset{\mathrm{VEV}}{=\!=} -\frac{v}{\sqrt{2}}\Big(\lambda^{ij}_{(d)}\,\bar{d}^{\,i}_L\,d^j_R + \lambda^{ij}_u\,\bar{u}^{\,i}_L u^j_R + \lambda^{ij}_e\,\bar{e}^{\,i}_L\,e^j_R\Big)$$

$\mathrm{SU}(3) \times \mathrm{U}(1)_{\mathrm{em}}$

Diagonalise the masses.

$$u^i_L \to U^i_{(u)\,j} u^j, \quad d^i_L \to U^i_{(d)\,j}\,d^j_L, \quad e_L \to U\,e_L$$

$$u^i_R \to W^i_{(u)j}\,u^j_R, \quad d_R \to W\,d_R, \quad e_R \to W\,e_R$$

$$\bar{u}_{L\,i}\not{\partial} u^i_L + \text{interactions}$$

$$\bar{u}_{R\,i}\not{\partial} u^i_R$$

$$\bar{d}_L \not{\partial} d_L$$

We want to diagonalise the mass terms without messing up the kinetic terms.

$$U^{\dagger}U = 1$$

$$\lambda^i_{d\,j}\,\bar{d}_{L\,i}\,d^j_R \rightarrow \left(U^{\dagger}\lambda W\right)^i_j\,\bar{d}_{L\,i}\,d^j_R$$

$$= \frac{\sqrt{2}}{v}\,m^i_d\,\delta^i_j \ \text{ no sum}$$

$$\text{“}\ \bar{\psi}\,\not{D}\psi\ \text{”} = \bar{L}^i_L\,\mathrm{i}\,\not{D}L^i_L + \bar{e}^{\,i}_R\,\mathrm{i}\,\not{D}\,e^i_R + \bar{Q}^{\,i}_L\mathrm{i}\,\not{D}Q^i_L + \bar{u}^{\,i}_R\,\mathrm{i}\,\not{D}u^i_R + \bar{d}^{\,i}_R\,\mathrm{i}\,\not{D}d^i_R =$$

$$= \text{kinetic}\,\not{\partial} + g\left(W^+_\mu J^\mu_{W^+} + W^-_\mu J^\mu_{W^-} + Z^0_\mu J^\mu_{Z^0}\right) + e\,A_\mu\,J^\mu_{\text{em}} + g_s\,G^A_\mu J^{A\mu}_{\text{QCD}}$$

$$J^\mu_{W^+} = \frac{1}{\sqrt{2}}\left(\bar{\nu}^{\,i}_L\gamma^\mu e^i_L + \bar{u}^{\,i}_L\gamma^\mu\,d^i_L\right)$$

$$\bar{Q}_L\mathrm{i}\,\not{D}Q_L = \bar{Q}_L\,\mathrm{i}\not{\partial}Q_L + W^\mu_+\bar{Q}_L\tau^+\gamma^\mu Q_L$$

$$\mathrm{D}_\mu = \ldots Z^0_\mu\left(\tau^3 - \sin^2\theta_Z\,Q\right)$$

$$J^\mu_{Z^0} = \frac{1}{\cos\theta_W}\sum_{\text{all }\psi\text{'s}}\ \bar{\psi}\gamma^\mu\left(\tau^3 - \sin^2\theta_W Q\right)\psi =$$

$$= \frac{1}{\cos\theta_W}\left(+\frac{1}{2}\bar{u}_L\gamma^\mu u_L + \bar{e}_L\gamma^\mu\left(-\frac{1}{2} + \sin^2\theta_W\right)e_L + \bar{e}_R\gamma^\mu\sin^2\theta_W\,e_R + \right)$$

$$J^{\text{em}}_\mu = -\,\bar{e}_{R,L}\gamma^\mu e_{L,R} + \frac{2}{3}\bar{u}\,\gamma^\mu u - \frac{1}{3}\,\bar{d}\gamma^\mu d$$

$$u^i_L \rightarrow U^i_{(u)\,j}\,u^j_L, \quad e_L \rightarrow U_{(e)}e_L$$

$$d_L \rightarrow U_{(d)}d_L$$

$$u_R \rightarrow W_{(u)}u_R, \quad e_R \rightarrow W_{(e)}e_R$$

$$J^\mu_{\text{em}} = -\,\bar{e}_{L\,i}\gamma^\mu e^i_L + \cdots \rightarrow \text{itself}$$

$$J^\mu_{Z_0} = \text{const} \times \bar{e}_{L\,i}\gamma^\mu e_L \rightarrow \text{itself}$$

$$J^\mu_{W^+} = \frac{1}{\sqrt{2}}\left(\bar{\nu}^{\,i}_L\gamma^\mu e^i_L + \bar{u}^{\,i}_L\gamma^\mu\,d^i_L\right) \rightarrow \frac{1}{\sqrt{2}}\left(\bar{u}^{\,i}_L\,\gamma^\mu U^{ij}_{(e)}e^j_L + \bar{u}^{\,i}_L\gamma^\mu U^{+ik}_{(u)}U^{kj}_{(d)}d^j_L\right)$$

$$V = U^{\dagger}_{(u)}U_{(d)}$$

$$V^{\dagger}V = U^{\dagger}_d U_u U^{\dagger}_u U_d = 1$$

General structure of V

Assume there are n families. (In reality: three.) V is a $\mathrm{U}(n)$ matrix.

$$\bar{u}^{\,i}_L V^j_i\,d_{L\,j}$$

$2n$ quarks. $2n$ phases. $V_{ij} \rightarrow \mathrm{e}^{\mathrm{i}\Delta\alpha_{ij}} V_{ij}$.

$\mathrm{U}(n)$, n^2. $\mathrm{SO}(n)$ matrix + phases. $\mathrm{SO}(n)$: $n(n-1)/2$. Phases $n(n+1)/2$. $2n-1$ of these phases can be eliminated, phases of quarks.

$$\begin{pmatrix} \cos\theta_c & \sin\theta_c \\ -\sin\theta_c & \cos\theta_c \end{pmatrix} \leftarrow \; n=2 \;\; \frac{2\times 3}{2} - (2\times 2-1) = 0$$
$$\begin{pmatrix} V_{ud} & V_{us} & V_{ub} \\ V_{cd} & V_{cs} & \ldots \\ \ldots & \ldots & V_{td} \end{pmatrix} \leftarrow \; n=3 \;\; \frac{3\times 4}{2} - (2\times 3-1) = 1\,\text{phase left}, \;\; CP\,\text{violation.}$$

Higgs couplings

$V(\phi)$ minimum at $\phi = \phi_0 = \frac{1}{\sqrt{2}}\begin{pmatrix} 0 \\ v \end{pmatrix}$

$$\mathcal{L}(\ldots, \phi_0) =$$

$$\phi = \begin{pmatrix} \phi_+ \\ \phi_0 \end{pmatrix}: \text{complex components}$$

$$\phi = \underbrace{U(x)}_{\mathrm{SU}(2)} \begin{pmatrix} 0 \\ \frac{1}{\sqrt{2}}(v + h(x)) \end{pmatrix}$$

U gauged. $h(x)$ remains, and corresponds to the Higgs boson.

$\mathcal{L}$: $v \rightarrow v + h$

Vector boson

$$m_W^2 = \frac{v^2 g^2}{2} W_\mu^+ W_\mu^- \rightarrow$$

$$\underbrace{\frac{v}{2}\lambda_{\mathrm{diag}}}_{m_\psi} \bar{\psi}_L \psi_R \rightarrow m_\psi \bar{\psi}_L \psi_R + \frac{m_\psi}{v} \bar{\psi}_L \psi_R h$$