2009–03–31

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σ: $p+p \rightarrow$ anything

i: $u, d, s, \bar{u}, \bar{d}, \bar{s}, g$.

$i+j \rightarrow$ anything. $\hat{\sigma}_{ij}(x_1 P_1, x_2 P_2)$ where P_1, P_2 is the proton 4-momentum.

$$\sigma = \sum_{ij} \int_0^1 \mathrm{d}x_1 \int_0^1 \mathrm{d}x_2 \; f_i(x_1)\, f_j(x_2)\, \hat{\sigma}_{ij}(x_1 P_1, x_2 P_2, s)$$

$$\frac{\mathrm{d}^2 \hat{\sigma}}{\mathrm{d}x_1 \mathrm{d}x_2}$$

$$\begin{array}{lcl} \mu_{\mathrm{R}} & \leftarrow & \text{Renormalisation scale} \\ \mu_{\mathrm{F}} & \leftarrow & \text{Factorisation scale} \end{array}$$

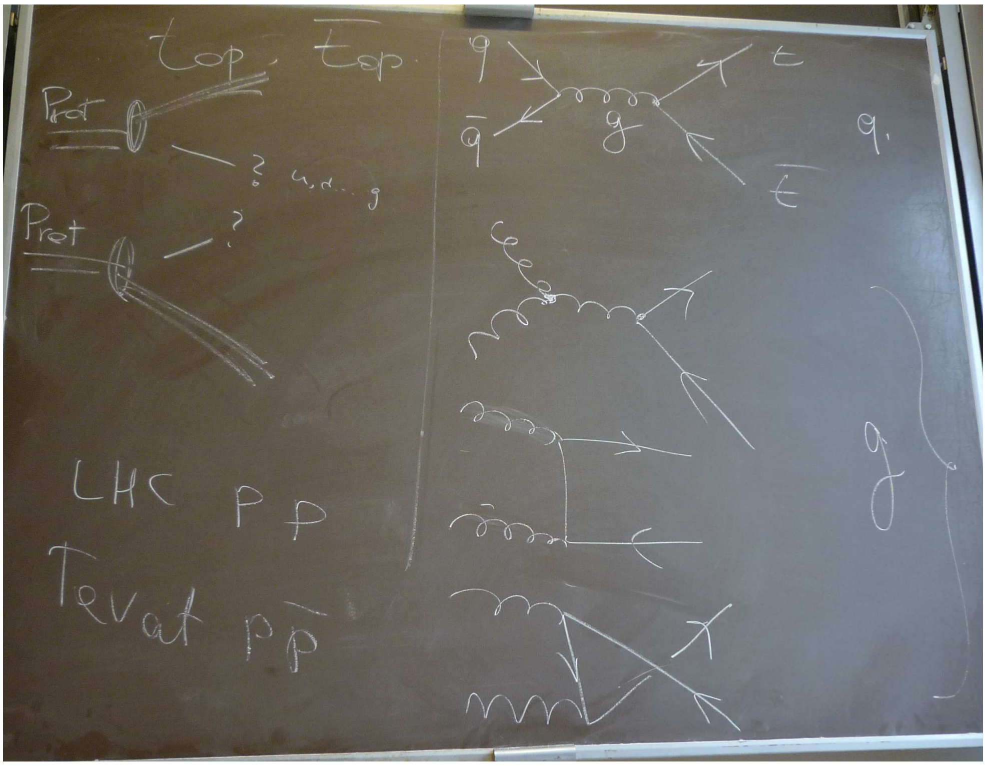

Figure 1.

Top decay $t \to W^+ + b$. Charge: $\frac{2}{3} \to +1 - \frac{1}{3}$. When you make a quark, you don't actually see a quark, you see a jet of particles. Here, a b jet. Every quark will be seen in the detector as a jet of particles.

The W^+ on the other hand can decay into $e^+ + \nu$, $\mu^+ + \nu$, $\tau^+ + \nu$ (10% each), or it could go into a quark–antiquark pair, some light quarks: $u\,\bar{d}$, $u\,\bar{s}$ (2 jets, 70%).

Dilepton:

$$t\,\bar{t} \to 2\,b\,(\text{jets}) + \mu^+ + \mu^- + \nu + \bar{\nu}$$

Hadron:

$$t\,\bar{t} \to 6\,\text{jets}$$

The Standard Model

Quarks (spin $\frac{1}{2}$)	$\begin{matrix} u & c & t & +\frac{2}{3} \\ d & s & b & -\frac{1}{3} \end{matrix}$	— Hadrons $\begin{cases} \text{Baryons: } 3q \text{ (and anti-)} \\ \text{Mesons: } q\,\bar{q} \end{cases}$
Leptons (spin $\frac{1}{2}$)	$e, \mu, \tau, \nu_e, \nu_\mu, \nu_\tau$	
Gauge bosons	$\gamma, W^\pm, Z_0$, 8 gluons	
Higgs	ϕ	

$$\mathcal{L} = F^2 + \bar{\psi}\,\not{D}\psi + (\mathrm{D}\phi)^2 + \bar{\psi}\phi\psi + \phi^2 + \phi^4$$

Gauge group: $\mathrm{SU}(3) \times \mathrm{SU}(2)_W \times \mathrm{U}(1)_Y$

If I have a Fermi field with Dirac spinor ψ I can construct a left-handed ψ_L or a right-handed ψ_R: $\psi_{L,R} = \frac{1}{2}(1 \pm \gamma_5)\psi = P_{L,R}\psi$ where $\gamma_5 = \mathrm{i}\,\gamma_0\,\gamma_1\,\gamma_2\,\gamma_3$.

$$P_L^2 = P_L, \quad P_R^2 = P_R, \quad P_L + P_R = 1, \quad P_L P_R = P_R P_L = 0$$

In the Standard Model the left-handed and right-handed components interact in different ways.

$$Q_L = \begin{pmatrix} u_L \\ d_L \end{pmatrix}$$

SU(2)	$s=$	0	$\frac{1}{2}$	1	...
	dim$=$	1	2	3	
		ψ	$\begin{pmatrix} \psi_1 \\ \psi_2 \end{pmatrix}$	$\begin{pmatrix} \cdot \\ \cdot \\ \cdot \end{pmatrix}$	
		singlet	doublet	triplet	
		•	□	□□	□□□

SU(3): □ triplet, $\bar{□}$ anti-triplet, □ = fundamental representation.

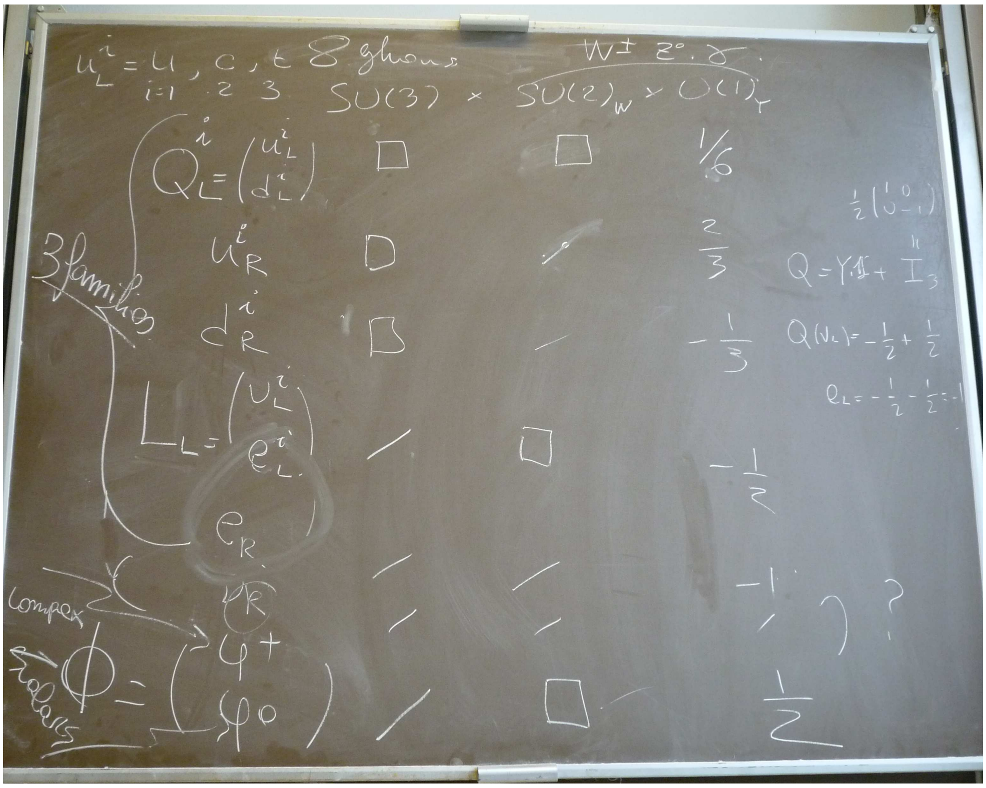

Figure 2. Table

$$\mathrm{SU}(N) = \{\text{Unitary } N \times N \text{ matrices of det}=1\}$$

$$g = \mathrm{e}^{\mathrm{i}A}, \quad g^{-1} = g^{\dagger} \Leftrightarrow A^{\dagger} = A \text{ Hermitian: Lie algebra}$$

$$\det(g) = 1 \Leftrightarrow \mathrm{tr}(A) = 0$$

It is much easier to work at the level of the Lie algebra, rather than the gauge group.

$\mathrm{SU}(2), 2\times 2$ Hermitian matrix, traceless. 3 matrices σ^i.

$$A = A^i \sigma^i \quad \text{where}\, \sigma^1 = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}, \ldots$$

$\mathrm{SU}(3), 3\times 3$ matrices. Gell–Mann matrices. 8 matrices.

$$\lambda^1 = \begin{pmatrix} 0 & 1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}, \quad \begin{pmatrix} 0 & -\mathrm{i} & 0 \\ \mathrm{i} & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix},$$

$$\lambda^3 = \begin{pmatrix} 1 & & \\ & -1 & \\ & & 0 \end{pmatrix}, \ldots, \quad \lambda^8 = \frac{1}{\sqrt{3}} \begin{pmatrix} 1 & & \\ & 1 & \\ & & -2 \end{pmatrix}$$

$$\left[\sigma^i, \sigma^j\right] = 2\,\mathrm{i}\,\varepsilon^{ijk} \sigma^k$$

$$\left[\lambda^A, \lambda^B\right] = 2\,\mathrm{i}\, f^{ABC} \lambda^C$$

where f^{ABC} is called the structure constant. All the properties of the Lie algebra is encoded into the structure constant.

$\tau^a = \frac{1}{2}\sigma^a$:

$$[\tau^a, \tau^b] = \mathrm{i}\,\varepsilon^{abc} \tau^c$$

$T^A = \frac{1}{2}\lambda^A$.

Gluon field G_μ where $\mu = 0, \ldots, 3$ is a 3×3 Hermitian matrix.

$$g_s, \quad \mathrm{SU}(3): \quad G_\mu = G_\mu^A T^A$$

$$g, \quad \mathrm{SU}(2): \quad A_\mu = A_\mu^a \tau^a$$

$$g', \quad \mathrm{U}(1): \quad B_\mu$$

$$\mathrm{D}_\mu Q_L = \partial_\mu Q - \mathrm{i}\, g_s\, G_\mu^A T^A Q_L - \mathrm{i}\, g\, A_\mu^a \tau^a Q_L - \mathrm{i}\, g' \frac{1}{6} B_\mu Q_L$$

$$Q^i_{L,\alpha,c,\xi}: \quad \begin{array}{ll} \alpha & \text{spinor index} \\ c = 1,2,3 & \text{colour index} \\ i & \text{flavour} \\ \xi = 1,2 & \text{up or down} \end{array}$$

A colour transformation

$$Q_c \rightarrow Q'_c = g_c^{c'}(x) Q_c(x)$$

where g is a $\mathrm{SU}(3)$ gauge transformation.

$$\mathrm{D}_\mu d_R = \partial_\mu d_R - \mathrm{i}\, g_s\, G_\mu^A T^A d_R - \mathrm{i} g' \left(-\frac{1}{3}\right) B_\mu d_R$$

$$\mathrm{D}_\mu e_R = \partial_\mu e_R - \mathrm{i}\, g' (-1) B_\mu d_R$$

Higgs:

$$\mathrm{D}_\mu \Phi = \partial_\mu \Phi - \mathrm{i}\, g\, A_\mu^a\, \tau^a \Phi - \mathrm{i}\, g' \cdot \frac{1}{2}\, B_\mu \Phi$$

$$\bar{\psi}\, \not{D} \psi = \sum_{i=1,2,3} \bar{Q}_L^{\beta c i \xi} \gamma_\beta^{\mu\alpha} (\mathrm{D}_\mu \Phi_L)^i_{\alpha,c,\xi}$$

$$\bar{Q}_L \not{D} Q_L + \bar{u}_R \not{D} u_R + \bar{d}_R \not{D}\, d_R + \bar{L}_L \not{D} L_L + \bar{e}_R \not{D} e_R$$

"F^2": Maxwell $\mathcal{L} = -\frac{1}{4}\, F_{\mu\nu} F^{\mu\nu}$ with $F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu$. Here $F_{\mu\nu} = \partial_\mu B_\nu - \partial_\nu B_\mu$:

$$\mathcal{L}_{\mathrm{SM}} = \underbrace{-\frac{1}{4}\, F_{\mu\nu}^A\, F^{A\mu\nu}}_{\mathrm{SU}(3)} \underbrace{-\frac{1}{4} F_{\mu\nu}^a F^{a\mu\nu}}_{\mathrm{SU}(2)} \underbrace{-\frac{1}{4} F_{\mu\nu} F^{\mu\nu}}_{\mathrm{U}(1)}$$

$$F_{\mu\nu}^A = \partial_\mu G_\nu^A - \partial_\nu G_\mu^A + \mathrm{i}\, g_s\, F^{ABC} G_\mu^B\, G_\nu^C$$

$$F_{\mu\nu} = F_{\mu\nu}^A T^A, \quad \mathrm{tr}(T^A T^B) = \frac{1}{2}\, \delta^{AB}$$

$$-\frac{1}{4}\, F_{\mu\nu}^A F^{A\mu\nu} = -\frac{1}{2}\, \mathrm{tr}(F_{\mu\nu} F^{\mu\nu})$$

$$\bar{Q}_L \not{D} Q_L, \quad \not{D} = \not{\partial} - \mathrm{i} g_s \not{G} - \mathrm{i}\, g\, \not{A} - \mathrm{i}\, g' \frac{1}{6}\, \not{B}$$

$$\bar{Q}_L \not{D} Q_L = \bar{Q}_L \not{\partial} Q_L + \cdots - \mathrm{i}\, g\, \bar{Q}_L \gamma^\mu \tau^a Q_L\, A_\mu^a$$

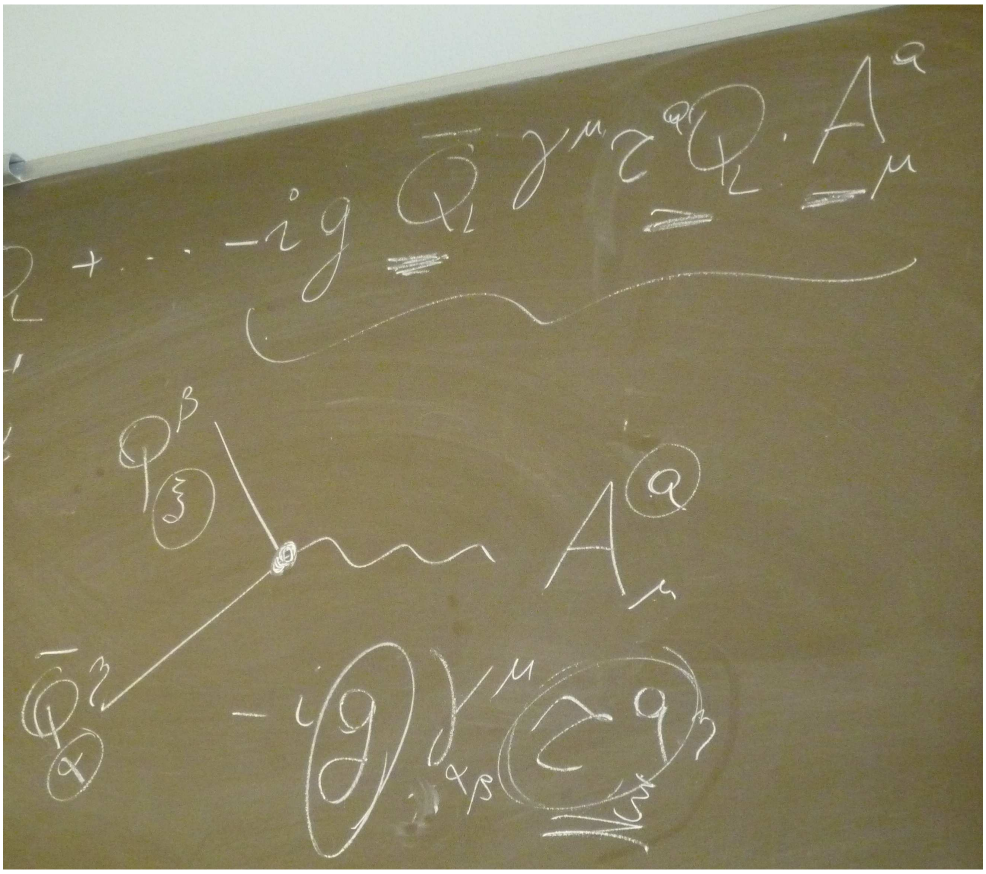

Figure 3.

Gluon: $F^2 = (\partial G)^2 + \; g_s G^2 \partial G + g_s^2 G^4$.

$$(\mathrm{D}\Phi)^2 = |\mathrm{D}_\mu \Phi|^2 = (\mathrm{D}^\mu \Phi)^\dagger (\mathrm{D}_\mu \Phi)$$

$m^2 B_\mu B^\mu \rightarrow$ as $B_\mu + \partial_\mu \chi$.