2009–03–25

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$$\sigma = \frac{\lambda^2}{64\pi s}$$

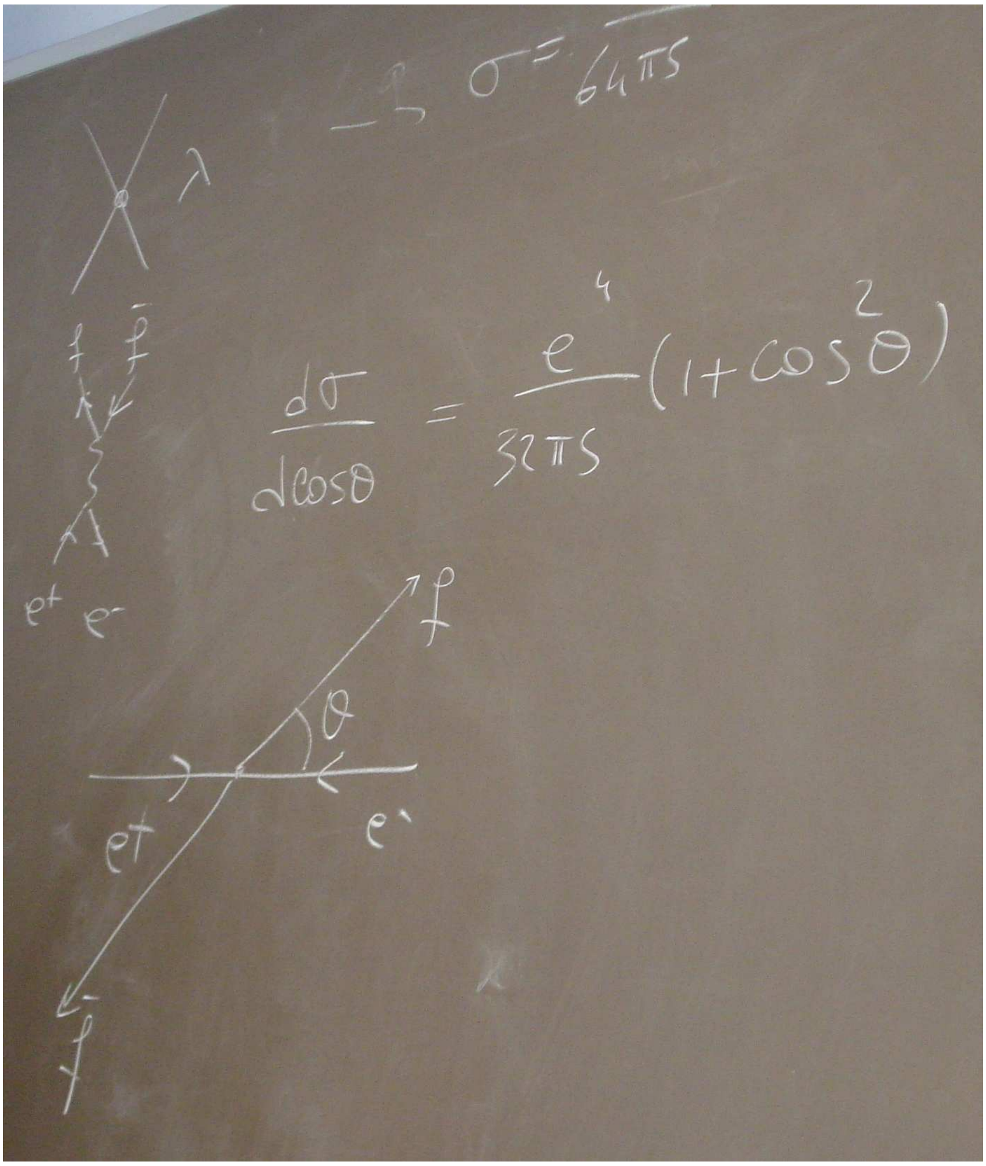

Figure 1.

$$\frac{\mathrm{d}\sigma}{\mathrm{d}\cos\theta} = \frac{e^4}{32\pi s}\left(1 + \cos^2\theta\right)$$

Basic aspects of high-energy physics. Feynman rules.

pp or $p\bar{p}$ collision.

Drell–Yan production is production of a (high-energy) lepton (μ^-, μ^+) pair in hadronic collisions; pp at LHC and $p\bar{p}$ at the Tevatron.

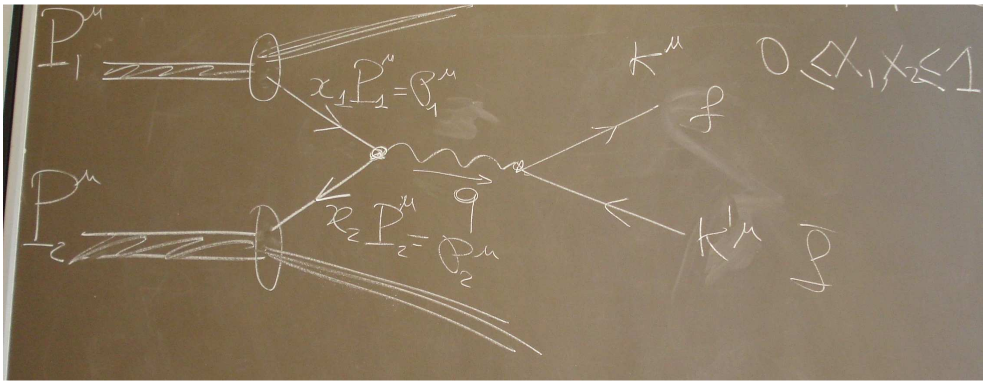

Figure 2. $0 \leqslant x_1 \leqslant 1, 0 \leqslant x_2 \leqslant 1$.

"That the proton consists of three quarks is a lie. It's a fairy tale that you tell little children so that they sleep well at night." The proton is more complicated.

$$i = u, d, s, \ldots, \bar{u}, \bar{d}, \bar{s}, \ldots, g$$

$f_i(x)\,\mathrm{d}x =$ probability of finding the "parton" i carrying a fraction x of the proton 4-momentum. PDF = Parton Distribution Function.

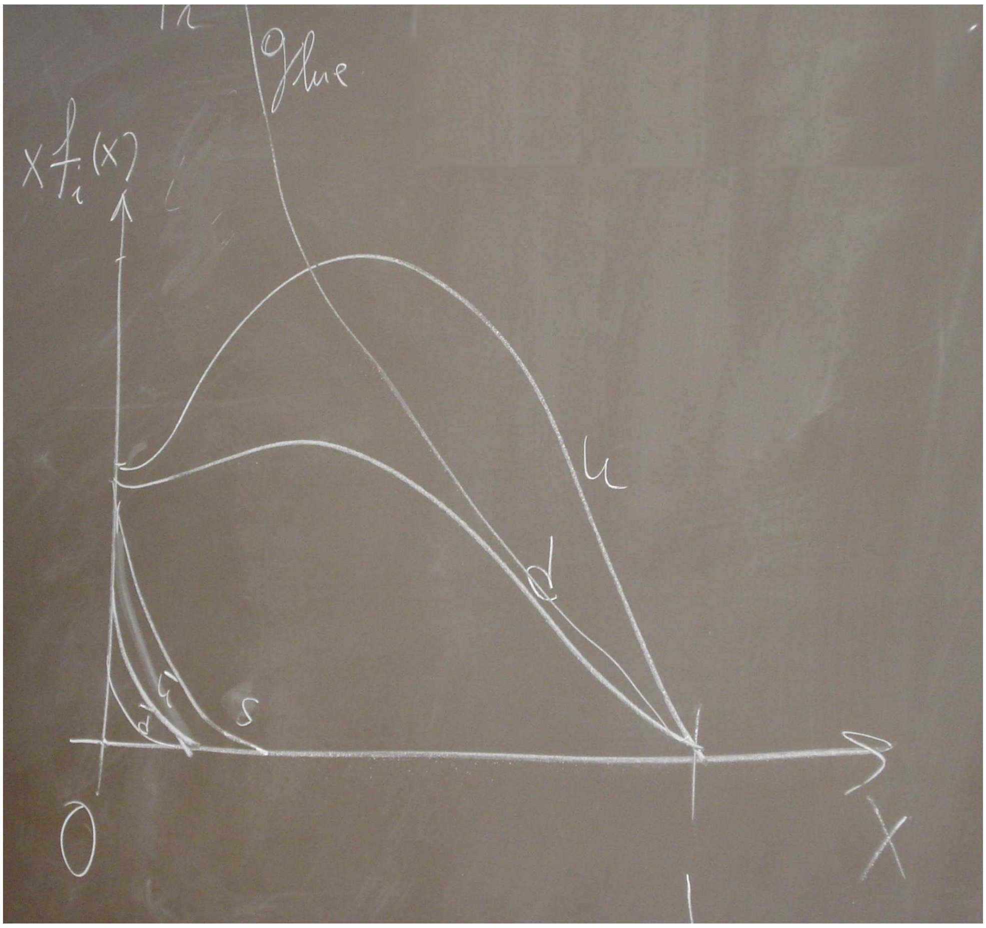

Figure 3.

$$\int_0^1 (f_u(x) - f_{\bar{u}}(x))\,\mathrm{d}x = 2, \quad \int_0^1 (f_d(x) - f_{\bar{d}}(x))\,\mathrm{d}x = 1, \quad \int_0^1 (f_s(x) - f_{\bar{s}}(x))\,\mathrm{d}x = 0.$$

For the neutron $\tilde{f}_d \approx f_u =$ pdf of u in the proton. (Equality appart from electromagnetic corrections.)

$$\int_0^1 \mathrm{d}x\, x(f_u(x) + \cdots + f_g(x)) = 1$$

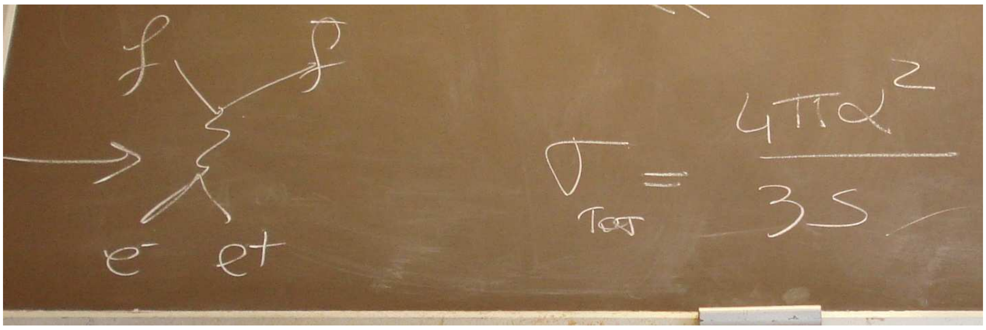

Figure 4. $\sigma_{\mathrm{tot}} = 4\pi\alpha^2/3s$ where $\alpha = e^2/4\pi$ is the fine structure constant

$$\sigma_{q\bar{q}} = \frac{1}{3} \cdot \frac{1}{3} \cdot 3 \cdot Q_q^2 \frac{4\pi\,\alpha^2}{3\,\hat{s}}$$

$$Q_u = Q_c = Q_t = \frac{2}{3}, \quad Q_d = Q_s = Q_b = -\frac{1}{3}$$

$$\hat{s} = (\wp_1 + \wp_2)^2 \simeq 2\,\wp_1 \cdot \wp_2 = 2\,x_1\,x_2\,P_1 \cdot P_2 \simeq x_1\,x_2 = S$$

Drell–Yan

$$\sigma(p\,p \to \mu^+\mu^- + X) = \int_0^1 \mathrm{d}x_1 \int_0^1 \mathrm{d}x_2 \sum_{u,d,s} f_q(x_1)\, f_{\bar{q}}(x_2)\, \frac{1}{3}\, Q_q^2 \frac{4\pi\alpha^2}{3\,x_1\,x_2\,S}$$

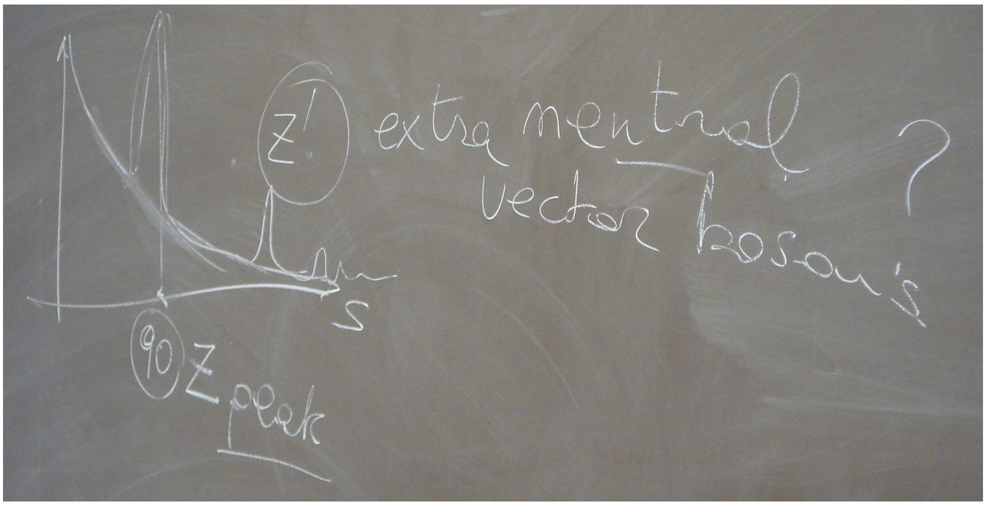

Figure 5. The mass of the Z boson gives a peak. If there are heavier Z-like bosons Z', they should show up in this spectrum. Various theories want an Z', but have not predicted its mass. Such a particle might be discovered at the LHC.

Instead of S, we measure $M^2 = (k + k')^2$, the inariant mass of the lepton pair. We want to replace x_1 and x_2 by M^2 and some other (what?) measurable thing.

$$\sigma = \int \mathrm{d}M^2 \frac{\mathrm{d}\sigma}{\mathrm{d}M^2}$$

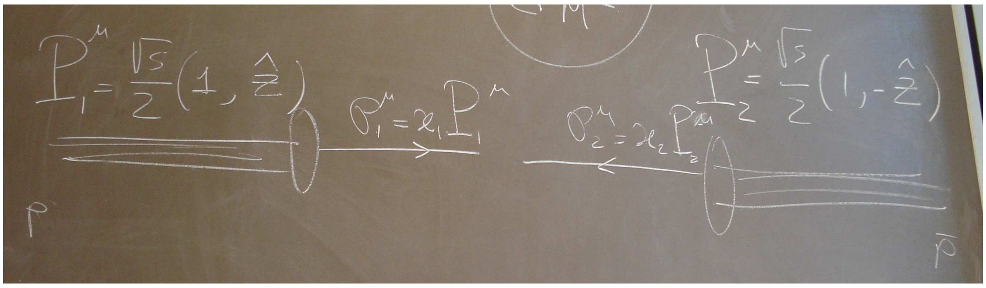

Figure 6.

$$\begin{pmatrix} \sqrt{\hat{s}}/2 \\ \sqrt{\hat{s}}/2 \end{pmatrix} = \begin{pmatrix} \cosh Y & -\sinh Y \\ -\sinh Y & \cosh Y \end{pmatrix} \begin{pmatrix} x_1\sqrt{s}/2 \\ x_1\sqrt{s}/2 \end{pmatrix}$$

The lab frame is the centre-of-mass frame of the protons, which is not the centre-of-mass frame of the quarks.

$$\begin{pmatrix} \sqrt{\hat{s}}/2 \\ -\sqrt{\hat{s}}/2 \end{pmatrix} = \begin{pmatrix} \cosh Y & -\sinh Y \\ -\sinh Y & \cosh Y \end{pmatrix} \begin{pmatrix} x_1\sqrt{s}/2 \\ -x_1\sqrt{s}/2 \end{pmatrix}$$

$$\sqrt{\hat{s}} = (\cosh Y - \sinh Y)\, x_1 \sqrt{s}$$

$$\sqrt{\hat{s}} = (\cosh Y + \sinh Y)\, x_2 \sqrt{s}$$

$$\frac{\cosh Y + \sinh Y}{\cosh Y - \sinh Y} = \frac{x_1}{x_2} \quad \Rightarrow \quad Y = \frac{1}{2}\log\frac{x_1}{x_2}$$

$$\begin{vmatrix} \frac{\partial Y}{\partial x_1} & \frac{\partial Y}{\partial x_2} \\ \frac{\partial M^2}{\partial x_1} & \frac{\partial M^2}{\partial x_2} \end{vmatrix} = \begin{vmatrix} \frac{1}{2\,x_1} & -\frac{1}{2\,x_2} \\ s\,x_2 & s\,x_1 \end{vmatrix} = S = \frac{M^2}{x_1\,x_2}$$

$$M^2 = x_1\,x_2\,S = \hat{s}$$

$$0 \leqslant x_1 = \mathrm{e}^{Y}\sqrt{\frac{M^2}{S}} \leqslant 1, \quad x_2 = \mathrm{e}^{-Y}\sqrt{\frac{M^2}{S}} \leqslant 1$$

The fact that $x_1, x_2 \leqslant 1$ forces

$$-\frac{1}{2}\log\frac{s}{M^2} \leqslant Y \leqslant +\frac{1}{2}\log\frac{S}{M^2}$$

$$\sigma = \int_0^s \mathrm{d}M^2 \underbrace{\int_{-\frac{1}{2}\log\frac{s}{M^2}}^{+\frac{1}{2}\log\frac{s}{M^2}} \mathrm{d}Y\, \frac{1}{M^2}\sum_q x_1 f_q(x_1)\, x_2\, f_{\bar{q}}(x_2)\, \frac{1}{3}\, Q_q^2\, \frac{4\pi\alpha^2}{3\,M^2}}_{\frac{\mathrm{d}\sigma}{\mathrm{d}M^2} \propto \frac{1}{M^4}}$$