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cat

cell $\sim 10^{-5}$ m

atom ~ 1 Å $= 10^{-10}\,\mathrm{m}$

nucleus $\sim 1\mathrm{fm} = 10^{-15}\,\mathrm{m}$

Factor of 100 000 between these.

$\rightarrow$ electron $< 10^{-19}$ m.

$c = 1, \hbar = 1.$

$$\boxed{c\,\hbar = 0.2\,\mathrm{GeV}\cdot\mathrm{fm}}$$

$m_p \sim 1\,\mathrm{GeV}$.

$m_e \sim 511\,\mathrm{keV} \sim 0.5\,\mathrm{MeV}$.

$m_\pi \sim 140\,\mathrm{MeV}$ correspnds to a length of order 1 fm and a time of order 10^{-23} s.

$m_p \sim 1\,\mathrm{GeV}$.

$E_{\mathrm{LHC}} \sim 14\,\mathrm{TeV} = 14\,000\,\mathrm{GeV}$.

$m_{W,Z} \sim 80 - 90\,\mathrm{GeV}$.

$m_t \sim 174\,\mathrm{GeV}$,

Cyclotron: $m v^2/4 = e\,B\,v$,

$$\omega_{\mathrm{cycl}} = \frac{v}{r} = \frac{e\,B}{m}$$

Assume nonrelativistic,

$$E_{\max} = \frac{1}{2}\,m\,v^2 = \frac{1}{2}\,m\,\omega^2\,r^2 = \frac{e^2\,B^2\,r^2}{2\,m}$$

$r = 1\mathrm{m}, B \sim 1\mathrm{T}, E = 4.5\,\mathrm{MeV} \ll 1\mathrm{GeV}$. Nonrelativistic: OK.

Reminder of special relativity: $\eta_{\mu\nu} \doteq \mathrm{diag}(+1, -1, -1, -1)$.

$|\boldsymbol{p}| = \wp$. $P^\mu = (E, \boldsymbol{p})$, $P^\mu P_\mu = P^2 = E^2 - |\boldsymbol{p}|^2 = m^2$

$$\boldsymbol{p} = \gamma\,m\,\boldsymbol{v}, \quad E = \gamma m, \quad \gamma = \frac{1}{\sqrt{1 - v^2}} = \frac{E}{m}\begin{cases} \simeq 1 & \text{nonrelativistically} \\ > & \text{relativistically} \end{cases}$$

$$|\boldsymbol{v}| = \beta = \frac{\wp}{E}$$

$$\mathrm{d}\tau^2 = \mathrm{d}t^2 - \mathrm{d}\boldsymbol{x}^2, \quad \mathrm{d}\tau = \mathrm{d}t/\gamma$$

$$\begin{array}{ll} \text{Non-relativistically} & \omega = \dfrac{e\,B}{m} \\ \text{Relativistically} & \omega = \dfrac{e\,B}{\gamma\,m} \end{array}$$

$$\frac{\mathrm{d}\boldsymbol{p}}{\mathrm{d}t} = e\,\boldsymbol{v} \times \boldsymbol{B}$$

$$\frac{\mathrm{d}P^\mu}{\mathrm{d}\tau} = e\,\frac{\mathrm{d}x_\nu}{\mathrm{d}\tau}\,F^{\mu\nu}$$

$$\boldsymbol{p} = \gamma\,m\,\boldsymbol{v} = \frac{1}{\sqrt{1 - v^2}}\,m\,\boldsymbol{v}$$

On the circular path with $|\boldsymbol{v}| = \text{constant}$

$$\gamma m \frac{\mathrm{d}\boldsymbol{v}}{\mathrm{d}t} = e\,\boldsymbol{v} \times \boldsymbol{B}, \qquad \left|\frac{\mathrm{d}\boldsymbol{v}}{\mathrm{d}t}\right| = \frac{v^2}{t}, \qquad \boldsymbol{B} \perp \boldsymbol{v}$$

$$\gamma_m \frac{v}{r} = e\,v\,B$$

$\omega = c/r$ is the maximal cyclotron frequency.

Radiation loss.

Larmer's formula:

$$\text{Power} = P = \frac{2\,e^2}{3\,c^3} \cdot a^2 = \frac{2\,\gamma^4\,e^2\,v^4}{3\,c^3\,r^2}, \qquad a = \left|\frac{\mathrm{d}\boldsymbol{x}}{\mathrm{d}\tau}\right|^2 = \gamma^2\,\frac{v^2}{r}$$

Nonrelativistic: $\gamma \simeq 1, E_{\text{kin}} \simeq \frac{1}{2}\,m\,v^2$

$$P \propto \left(\frac{E_{\text{kin}}}{m}\right)^2 \cdot \frac{1}{r^2}$$

Ultrarelativistic: $\gamma = E_{\text{tot}}/m c^2 \equiv E_{\text{kin}}/m c^2$, $v = c$. Putting this into the formula for P you realise to your great disappointment that...

$$P = \left(\frac{E_{\text{kin}}}{m}\right)^4 \frac{1}{r^2}$$

You loose more and more power when you increase the energy, with a power 4! Bad when accelerating light particles.

Two main parameters:

- Centre-of-mass energy: $(p_1 + p_2)^2 = s = E_{\text{cm}}^2$.

Fixed target experiments: $P_1^\mu = (E_{\text{lab}}, \wp\boldsymbol{e}_z)$, $P_2^\mu = (m, \boldsymbol{0})$.

$$s = (P_1 + P_2)^2 = (E_{\text{lab}} + m)^2 - \wp^2 = E_{\text{lab}}^2 + 2\,m\,E_{\text{lab}} + m^2 - \wp^2$$

$(E_{\text{lab}}^2 = \wp^2 + m^2)$

$$s = E_{\text{cm}}^2 \sim 2\,m\,E_{\text{lab}}$$

- The Luminosity, $\mathcal{L}$. $[\mathcal{L}] = L^{-2}T^{-1}$ (flux). $\mathcal{L}_{\text{LHC}} \sim 10^{34}\,\text{cm}^{-2}\,\text{s}^{-1}$.

Let us denote by N the number of events per second.

$$N = \mathcal{L}\,\sigma$$

where σ is the total cross section.

$$\frac{\mathrm{d}N}{\mathrm{d}E} = \mathcal{L}\,\frac{\mathrm{d}\sigma}{\mathrm{d}E}$$

Tevatron, started operating 1983, is colliding $p + \bar{p}$ with $E_{\text{cm}} \sim 2\,\text{TeV}$ with $\mathcal{L} \simeq 10^{32}\,\text{cm}^{-2}\,\text{s}^{-1}$.

Lep 1989: $e^+ + e^-$, $E_{\text{cm}} \sim 200\,\text{GeV}$. $\mathcal{L} \approx 10^{32}\,\text{cm}^{-2}\,\text{s}^{-1}$.

LHC 2009: $p + p$, $E_{\text{cm}} \sim 14\,\text{TeV}$, $\mathcal{L} \approx 10^{34}\,\text{cm}^{-2}\,\text{s}^{-1}$.