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Anisotropies in the CMBR

The CMBR provides strong support for the Big Bang scenario. However, the isotropy in the CMBR is in strong contrast to what we observe today: matter is lumped together in galaxies and the temperature and density of a galaxy is vastly different from the space between galaxies. The structure of the matter distribution that we observe today is formed by gravitational instability. Small deviations from homogeneity grow due to gravitational attraction and eventually form stars and galaxies.

Anisotropies in the CMBR

The observation of small anisotropies ($\mathcal{O}(10^{-5})$) in the CMBR by COBE (and later by WMAP) supports this idea of structure formation since at the time of decoupling it can be shown that

$$\frac{\delta\rho}{\rho} \sim \frac{\delta T}{T}$$

The Sachs–Wolfe effect

There is a relation between the temperature fluctuations and the gravitational potential. In the Newtonian approximation the metric is

$$\mathrm{d}s^2 = (1 + 2\Phi)\,\mathrm{d}t^2 - (1 - 2\,\Phi)\left(\mathrm{d}x^2 + \mathrm{d}y^2 + \mathrm{d}z^2\right)$$

Here Φ is the Newtonian potential (outside a spherical object, e.g. a star, $\Phi = -\,G\,M/r$). One can show that $\delta T/T + \Phi$ is constant in the above geometry (by "energy conservation" or by analysing geodetic motion.) This is really only true to zeroth order and there are corrections to this (integrated Sachs-Wolfe effect) but we will not consider them.

We have

$$\left(\frac{\delta T}{T}\right)_{\mathrm{observed}} = \left(\frac{\delta T}{T}\right)_{\mathrm{emitted}} + \Phi_{\mathrm{emitted}}$$

(Assuming $\Phi_{\mathrm{observed}} = 0$.) We know (e.g. from entropy conservation) that $a\,T = \text{constant}$. This implies

$$\frac{\delta T}{T} = -\,\frac{\delta a}{a}.$$

For a matter dominated universe $a(t) \sim t^{2/3}$ which implies $\delta a/a = \frac{2}{3}\,\delta t/t$. Now $\mathrm{d}\tau = \sqrt{1 + 2\Phi}\,\mathrm{d}t \approx (1 + \Phi)\mathrm{d}t \equiv \left(1 + \frac{\delta t}{t}\right)\mathrm{d}t \Rightarrow \frac{\delta t}{t} \approx \Phi$. Combining the above results we find

$$\left(\frac{\mathrm{d}T}{T}\right)_{\mathrm{emitted}} = -\,\frac{2}{3}\,\Phi_{\mathrm{emitted}}$$

and finally,

$$\left(\frac{\mathrm{d}T}{T}\right)_{\mathrm{observed}} = \frac{1}{3}\,\Phi_{\mathrm{emitted}}.$$

Thus the fluctuation in temperature measured today allows us to determine the gravitational potential at the time when the photons decoupled. How large is the angle subtended today by a causally connected region at the time of decoupling? We saw before that such a region is of size $\sim 1/H(z = z_{\mathrm{dec}})$. Thus

$$\Delta\theta = \frac{1}{d_A}\,\frac{1}{H}$$

where d_A was defined in chapter four.

Fluctuations in the CMB

Any function $f(\theta,\varphi)$ on the sphere can be expanded in spherical harmonics.

$$f(\theta,\varphi)=\sum_{l=0}^{\infty}\sum_{m=-l}^{l}a_{lm}Y_{lm}(\theta,\varphi)$$

In particular, the temperature fluctuations $\delta T/T$ can be expanded as:

$$\frac{\delta T}{T}=\sum_{l=2}^{\infty}\sum_{m=-l}^{l}a_{lm}\,Y_{lm}(\theta,\varphi)$$

(Here $l=0$ corresponds to the isotropic piece and is hence excluded; $l=1$ is the dipole piece and is also excluded since it is affected by our relative motion compared to the cosmic rest frame.)

Observationally, from a measured set of a_{lm} we can form the angular average c_l:

$$c_l=\langle a_{lm}\,a_{lm}{}^{*}\rangle=\frac{1}{2\,l+1}\sum_{m=-l}^{l}a_{lm}\,a_{lm}^{*}$$

The set of c_l's is the basic information containing set of the microwave background. One draw-back is that the statistical properties gathered from a single vantage point is limited by "cosmic variance". Since there are only $2l+1$ independent a_{lm} the variance

$$\frac{\delta c_l}{c_l}\approx(2l+1)^{-1/2}$$

These c_l's have been measured by WMAP.

The best fit to the data is the so called ΛCDM model. CDM stands for "cold dark matter". This is a model with $\Omega_M\approx0.3$ and $\Omega_\Lambda=0.7$, $\Omega=\Omega_M+\Omega_\Lambda\approx1$.

Cosmic variance can be reduced by going to large l (small angles) but then other problems arise since at small angles the evolution of the universe becomes important.

Alternatives to the inflation scenario (Very speculative — probably wrong.)

Problems with the standard setup:

- It is not explained what came before inflation.
- Dark matter/energy require separate explanation.

A model inspired by string theory:

Two string theory ingredients:

1) Extra dimensions. In string theory more than four space time dimensions.

2) Branes. In string theory there are higher-dimensional objects. 1-brane = string. 2-brane = membrane.

It has been suggested that our universe is a 3-brane embedded in a higher-dimensional space (brane-world scenario). Quarks, leptons etc are on the brane. Only gravity "lives" in the full higher-dimensional space.

For simplicity 3-brane in $d=5$. In the five-dimensional space there can be several branes:

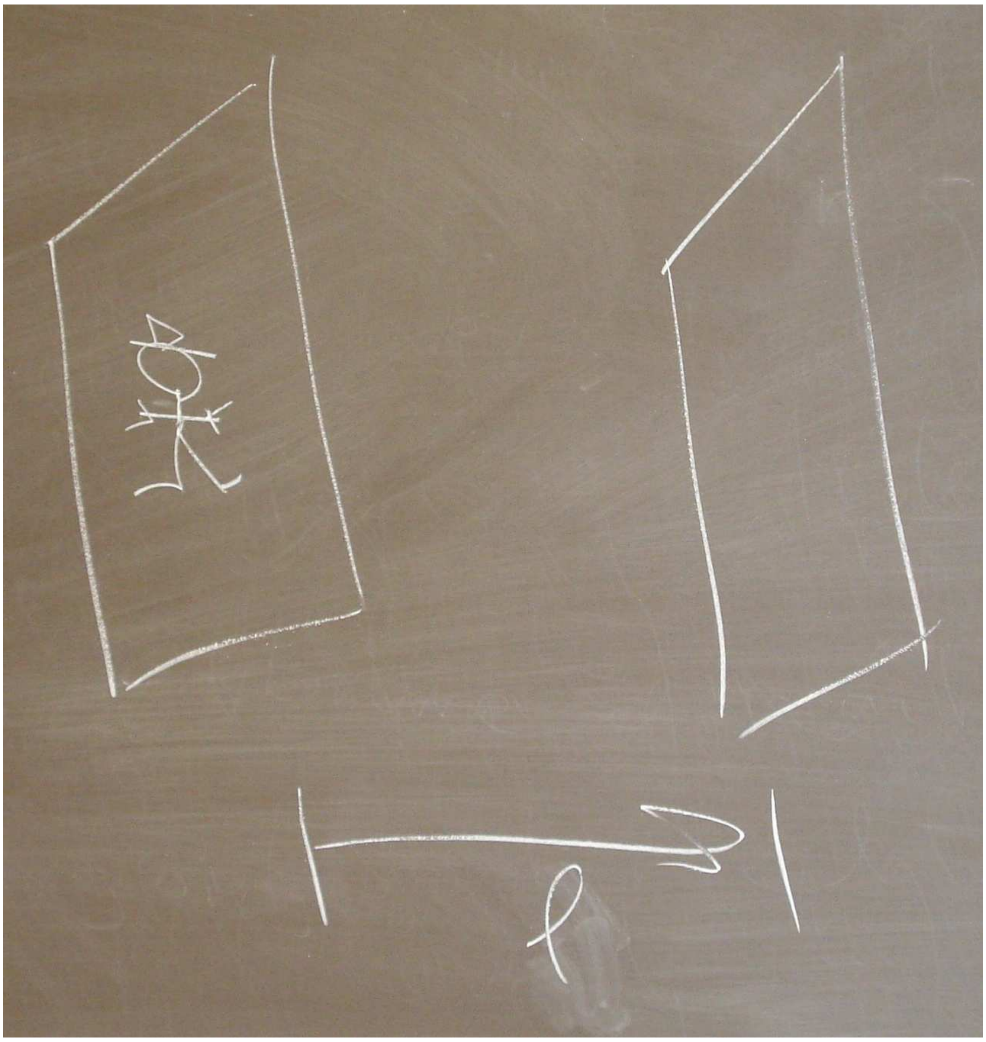

Figure 1. We are on one brane. There can be other branes, that we don't see.

It is of interest to consider what happens when branes collide. Big Bang $\rightarrow$ collision of branes? For this to work, we need a cyclic model.

Different predictions compared to inflation: could possibly be observed by Planck.

Drawbacks:

- Bounce is not understood at all.
- Potential ad hoc, not understood at all.
- etc.

Some good features:

- Dark energy could be force between branes.
- Dark matter could live on our brane *or* on the hidden brane.
- Brane setup seemingly avoids the problems with previous attempts at cyclic models.