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Lecturer: Niclas Wyllard

Last time we discussed a number of problems.

A solution: inflation

To solve the horizon problem we must allow points that are outside the horizon today to have been inside the horizon at some point in the past. This means that there must have been a period in the history of the universe when the expansion rate was more rapid than the rate at which the horizon grew. If this is so then regions which are not in causal contact today could have been so in the past. From the previous discussion the only possibility for this to occur is if $\rho + 3p < 0$. This is inflation: a period when the normally attractive force of gravity becomes repulsive resulting in an explosive expansion of the universe.

Inflation also solves the flatness problem since during inflation Ω will move towards 1 (rather than away from it).

Since inflation involves such an explosive expansion, the large increase in $a(t)$ will mean that any matter density will rapidly drop to near zero. Thus any future evolution of the universe is essentially independent of what happened before inflation.

One possible way to have inflation is if $p = -\rho$, since then $\rho + 3p = -2p < 0$. But also $\dot{\rho} = -3H(\rho + p) = 0$. Thus energy density is constant (need energy to "recreate" matter after inflation.) As we saw before $p = -\rho$ arises if $T_{\mu\nu} = \Lambda\, g_{\mu\nu}$ (Λ cosmological constant).

Most models of inflation involve a scalar field ϕ (the inflaton) with

$$T_{\mu\nu} = \partial_\mu\phi\, \partial_\nu\phi - g_{\mu\nu}\left(\frac{1}{2}\partial_\rho\phi\ \partial^\rho\phi - V(\phi)\right)$$

Comparing this to the above we see that inflation will occur if at some point $T_{\mu\nu}$ is dominated by the potential energy $V(\phi)$ of ϕ.

How did the universe end up in a state where the potential energy of the scalar field ϕ dominates the total energy density? Several possibilities have been discussed in the literature.

1) Old inflation (Guth)

In this scenario one assumes that the scalar field got stuck in a metastable (false) vacuum at $\phi = 0$ during a first order phase transition. During the phase transition ϕ will change its value from zero to a non-zero value.

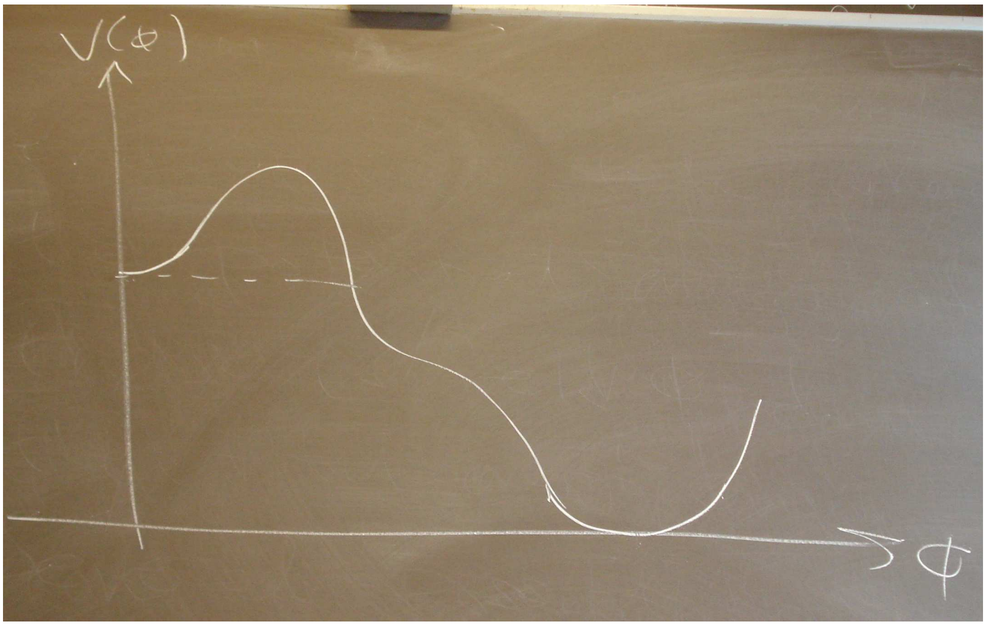

Figure 1. The potential, schematically.

If there is an energy barrier between the two vacua then the transition has to occur via quantum mechanical tunneling. Since the tunneling process is slow one can easily get lots of inflation. During the phase transition bubbles of the new phase are formed. There is a serious problem with this scenario in that the bubbles do not collide, so the universe ends up in an inhomogeneous state. There is "graceful exit" into the standard FRW universe.

2) New inflation

This scenario is based on a potential of the form (cf. figure 2).

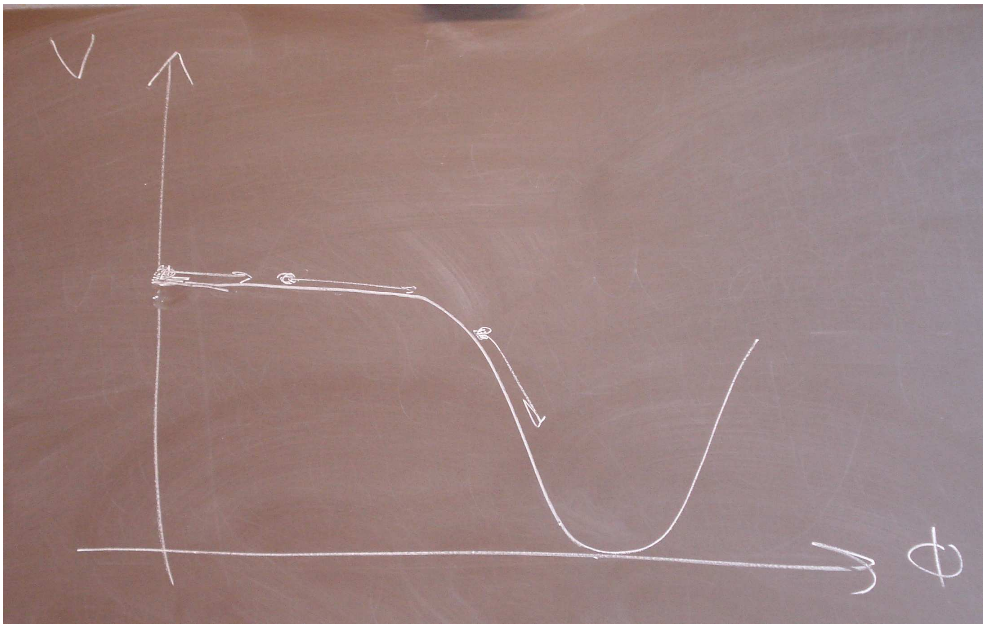

Figure 2. Potential $V(\phi)$ in New Inflation.

Because the potential is very flat and has a maximum at $\phi = 0$ the scalar escapes from $\phi = 0$ not via tunneling but due to quantum fluctuations. It then rolls slowly towards the global minimum. During the slow-roll the evolution of ϕ is purely classical. During the time it takes ϕ to evolve to the new vacuum the universe has an enormous vacuum energy $\rho \sim V(\phi)$ and expnads exponentially. As ϕ nears the minimum the potential steepens and starts to oscillate around the minimum (ϕ invevitably overshoots the minimum). Particle creation (decay of ϕ into other particles) acts as friction and will dampen the oscillations.

3) Chaotic inflation (Andrei Linde)

This is then name given to the most general class of models with slow-roll behaviour.

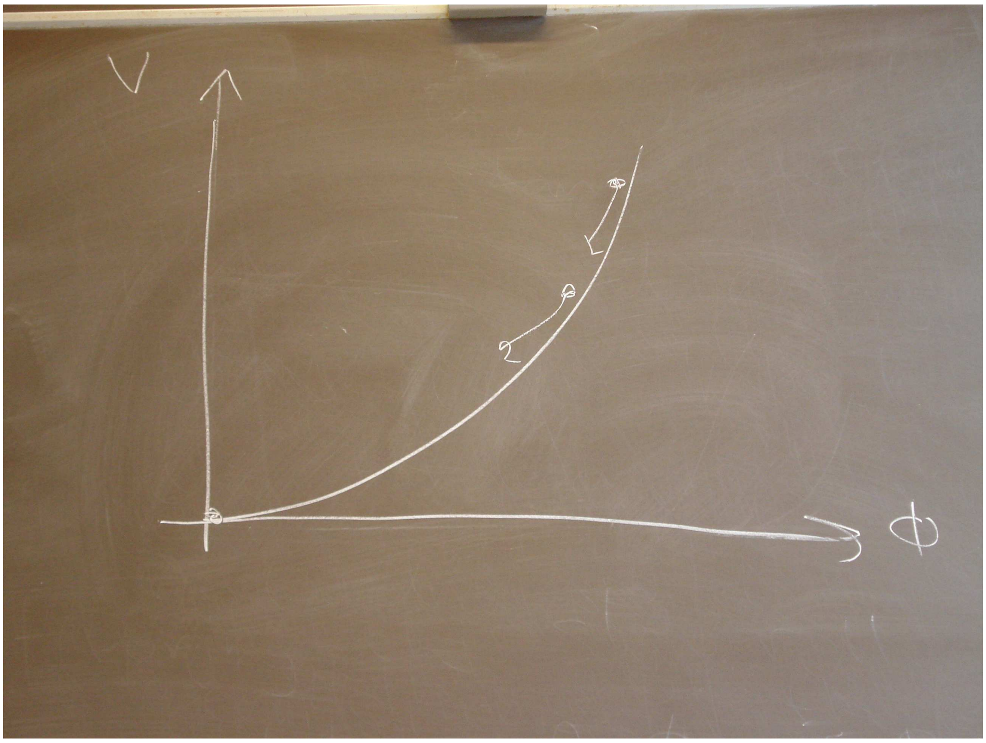

Figure 3.

The name chaotic is related to the possibility of having almost arbitrary initial conditions.

During the classical motion the equations of motion are easy to obtain. The energy density and pressure assuciated with the above $T_{\mu\nu}$ are

$$\begin{cases} \rho = \frac{1}{2}\dot{\phi}^2 + V(\phi) \\ p = \frac{1}{2}\dot{\phi}^2 - V(\phi) \end{cases}$$

(Here we neglected the spatial derivatives.) The equation of motion for ϕ is

$$\ddot{\phi} + 3H\dot{\phi} + V'(\phi) = 0$$

supplemented by the Friedmann equation:

$$H^2 + \frac{k}{a^2} = \frac{8\pi G}{3}\left(\frac{1}{2}\,\dot{\phi}^2 + V(\phi)\right)$$