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Freeze-out and dark matter

What happens when a particle species goes out of equilibrium? Let's consider a case of great interest to the dark matter problem. Assume there exists a particle χ with an antiparticle $\bar{\chi}$ such that they can annihilate or be pair-created according to $\chi + \bar{\chi} \leftrightarrow Y + \bar{Y}$, where $Y, \bar{Y}$ can be quarks, leptons... and are assumed bo be in thermal equilibrium with the photons and other light particles in the early universe.

The equation which governs the departure of the number density n_χ $(= n_{\bar{\chi}})$ of ξ from equilibrium is

$$\frac{\mathrm{d}}{\mathrm{d}t} n_\chi + 3\, H\, n_\chi = -\, \langle \sigma_{\text{annihilation}}\, |\boldsymbol{v}| \rangle \left[n_\chi^2 - \left(n_\chi^{\text{Eq}} \right)^2 \right]$$

("Boltzmann equation" from non-equilibrium thermodynamics.) $\sigma_{\text{annihilation}}$ is the toootal annihilation cross section of $\chi + \bar{\chi} \to \text{stuff}$. $\langle \cdots \rangle$ denotes a thermal average and $\boldsymbol{v}$ is the velocity. n_χ^{Eq} is the equilibrium density. Using

$$t = 0.3 \frac{m_{\text{Pl}}}{T^2 \sqrt{g_{\text{eff}}}}$$

and changing variables to $x = m_\chi / T$ and $Y_\chi = n_\chi / s$, where s is the entropy density, one can show

$$\frac{x}{Y_\chi^{\text{Eq}}} \frac{\mathrm{d}Y_\chi}{\mathrm{d}x} = -\frac{\Gamma}{H} \left[\left(\frac{Y_\chi}{Y_\chi^{\text{Eq}}} \right)^2 - 1 \right]$$

This equation can be solved numerically with boundary condition $Y \simeq Y^{\text{Eq}}$ at small x (since at high T the χ's were in thermal equilibrium with the other particles.) Note that Γ / H enters into the equation and determines the evolution.

A special case is if the species χ was relativistic at freeze out when $Y_\chi(\infty) = Y_\chi^{\text{Eq}}(x_f) = [x_f = m_\chi / T_f] =$

$$= \frac{45\, \zeta(3)}{2\pi^4} \frac{g_{\text{eff}}}{g_{\text{eff}}^s(x_f)}$$

where $g_{\text{eff}} = g$ for bosons, and $g_{\text{eff}} = \frac{3}{4}\, g$ for fermions. Such a particle is called a hot relic. The present mass density of a hot relic can be show to be $\Omega_\chi\, h^2 = 7.8 \times 10^{-2} \frac{g_{\text{eff}}}{g_{\text{eff}}^s(x_f)} \left(\frac{m_\chi}{1\,\text{eV}} \right)$.

Example: For a neutrino one has $g_{\text{eff}} = 2 \times \frac{3}{4} = \frac{3}{2}$ and $g_{\text{eff}}^s = 10.75$.

$$\begin{array}{ccccccccc} 10.75 & = & 2 & + \frac{7}{8} & (\; 2 \times 2 & + & 2 \times 2 & + & 2 \;) \\ & & \gamma & & e^- & & e^+ & & \nu \end{array}$$

Demanding that $\Omega_{\nu\bar{\nu}}\, h^2 < 1$ one finds:

$$\sum_{i=1}^{3} m_{\nu_i} < (90\,\text{eV})\, \Omega_M\, h^2$$

Thus there is a bound on the neutrino masses coming from cosmology!

One dark matter candidate is the so called neutralino (a particle in supersymmetric models). It is electrically neutral and does not emit or absorb radiation. A mass arund 30 GeV — few TeV could give a realistic contribution to Ω_M.

Another dark matter candidate is te so called axion, a light boson. A mass of $10^{-6} - 10^{-3}$ could give a sizable contribution to Ω_M.

Inflation

As we have discussed the hot big bang model is very successful. It can be used to describe the universe from at least $t = 10^{-2}$ s until today $t = 14$ Gyr. There are essentially no observational data which are in conflict with the standard model of cosmology. Nevertheless, there are some problematic aspects of the model which have to do with initial conditions.

The horizon problem (homogeneity, isotropy problem)

Why is the universe we observe today so homogeneous and isotropic?

Recall that regions in causal contact are bounded by horizons. Let's compare the size of a horizon $\sim 1/H$ witht he scale factor a. We have

$$\frac{a}{\frac{1}{H}} = \dot{a}$$

From the Friedmann equations we know

$$\frac{\ddot{a}}{a} = -\frac{4\pi G}{3}(\rho + 3p)$$

Under normal circumstances, $\rho > 0, p > 0$, which implies $\ddot{a} < 0$. This means that $\dot{a}$ decreases with time, which in turn means that the horizon size grows faster than the scale factor. If two points are outside the horizon today they have always been outside the horizon.

If we look at diametrically opposite points on the sky we are looking at points which have not yet come into causal contact (the CMBR just reached us, and we are in the middle). Yet the temperatures of the two regions are equal to a very high accuracy. How did regions which were never in causal contact turn out to have the same temperature? This is the horizon problem.

One can solve it by imposing suitable initial conditions. However, if one looks closer at the problem one comes to the conclusion that at the Planck time one must require that the universe to be homogeneous over $\sim 10^{83}$ causally disconnected regions. This is very unnatural.

The flatness problem

From the Friedmann equations we know

$$\Omega = \frac{\rho}{\rho_{\text{crit}}} = 1 + \frac{k}{\dot{a}^2} = \frac{1}{1-x}$$

where

$$x = \frac{3k}{8\pi G\,\rho\, a^2}$$

$$x \sim \begin{cases} a^2 & \text{for radiation} \\ a & \text{for matter} \end{cases}$$

Above we argued that $\dot{a}$ decreases with time. This means that Ω deviates more and more from 1. Now, if Ω has been deviating more and more from 1 for 10 billion years or so, why is it still so close to 1? For example, one can argue

$$|\Omega(t_{\text{Pl}}) - 1| \lesssim 10^{-60}$$

is required to get the value of Ω we observe today.

The monopole problem

AS we discussed briefly before monopoles are possible solutions in (extensions of) the standard model of particle physics. These are heavy particles, which feel an urge to dominate the universe. There is no way to suppress such particles if they were produced in the early universe.