2009–02–18

Lecturer: Niclas Wyllard

Example. Assume that there exists a particle X and its antiparticle $\bar{X}$ such that the following decays are possible:

(q is a quark, l is a lepton)	Probability (branching ratio)	Baryon number of final state
$X \to q\, q$	r	$2/3$
$X \to \bar{q}\, l$	$1-r$	$-1/3$
$\bar{X} \to \bar{q}\, \bar{q}$	$\bar{r}$	$-2/3$
$\bar{X} \to q\, \bar{l}$	$1-\bar{r}$	$1/3$

If C was not violated, we would have $r = \bar{r}$. ($q \to \bar{q}$ under C). The same is true for CP.

Both C and $C\,P$ violating processes are present in the standard model but it seems some new physics is needed to get the right amount of baryon–antibaryon asymmetry.

> "Baryogenesis provides a compelling argument that there is some kind of quark-lepton unification."

Recombination and the Saha equation

Incorporating the matter–antimatter asymmetry we have to a good approximation $n_{e^+} = n_{\bar{p}} = 0$ and $n_{e^-} = n_p$ (after $e^+e^- \to 2\gamma$ annihilation has taken place).

The interaction between electrons and photons is dominated by Thomson scattering $e^- + \gamma \to e^- + \gamma$ which has $\sigma_T \sim \alpha^2/m_e^2$ so

$$\Gamma_\gamma = n_e\, c\, \sigma_T = n_e\, \sigma_T$$

when $\Gamma_\gamma < H$ the photons will decouple. This happens around $T \sim 1 - 10$ eV. To calculate n_e one needs to to [*sic*] take into account that electrons can combine with protons to form hydrogen plus photons. (The opposite process is also possible.)

$$p + e^- \rightleftarrows \mathtt{H} + \gamma$$

implies that $\mu_p + \mu_e = \mu_{\mathtt{H}}$ ($\mu_\gamma = 0$). Charge neutrality: $n_e = n_p$. Since $T \lesssim 10$ eV, e, p and $\mathtt{H}$ are all non-relativistic and hence

$$n_i = g_i \left(\frac{m_i T}{2\pi}\right)^{3/2} \exp\left\{\frac{\mu_i - m_i}{T}\right\}, \quad i = e, p, \mathtt{H}.$$

Using $m_{\mathtt{H}} = m_e + m_p - B$, where B is the binding energy of hydrogen ($B = 13.6$eV), plus the above results one can show that

$$X_e = \frac{n_p}{n_p + n_{\mathtt{H}}}$$

is related to

$$\eta_B = \frac{n_p + n_{\mathtt{H}}}{n_\gamma}$$

as

$$\frac{1 - X_e}{X_e^3} = \frac{4\sqrt{2}\,\zeta(3)}{\sqrt{\pi}}\, \eta_B \left(\frac{T}{m_e}\right)^{3/2} \mathrm{e}^{B/T}$$

(Saha equation)

The baryon to photon ration η_B can be calculated using previous results:

$$\eta_B = \frac{n_B}{n_\gamma} = \frac{\rho_M/m_N}{n_\gamma} \approx 2.7\times 10^{-8}\,\Omega_B\,h^2$$

Nucleosynthesis constrains $\Omega_B\,h^2$ to be ~ 0.02. Using this value to solve for the redshift ($T = 2.73(1+z)$) when the ionisation fraction drops below 10% one finds that this occurs around $z_{\text{rec}} \sim 1300$. This is called recombination.

$$T_{\text{rec}} = 2.73(1+z_{\text{rec}}) \simeq 0.3\,\text{eV}$$

Thermodynamical equilibrium can only be maintained down to about $z\sim 1100$ (when $\Gamma \sim H$ for $e^- + p \to \mathtt{H} + \gamma$). After that the ionisation fraction is frozen. However, the mean free path of the photons due to this ionisation fraction is on the order of magnitude of the size of the visible universe. The region around $z_{\text{dec}} \sim 1100$ is called the surface of last scattering of the CMB photons. The photon recombination and decoupling is one of the most important epochs in the early universe since this is when the universe became transparent to optical photons. The photons still still [*sic*] travel through the universe today: CMBR.

Nucleosynthesis

The fact that the universe is composed of 76% H, 24% He plus small amounts of ^{3}He, D (^{2}H) and ^{7}Li is successfully reproduced by the Big Bang model (Big Bang nucleosynthesis). Using numerical methods one can follow (accurately) the number densities of the various light elements from the first few seconds after the Big Bang when they were first synthesised and onwards. It is quite striking that all the densities (He, ^{3}He, D, ^{7}Lie) agree with experiments since they depend on one parameter: η_B — the baryon to photon ratio.

Furthermore, the experimental results give an upper limit to η_B and hence to $\Omega_B h^2$ (~ 0.02). Schematic outline of nucleosynthesis: $t \ll 1\,\text{s}$ ($T \gg 1\,\text{MeV}$):

Neutrinos, electrons and positrons were in equilibrium through the weak interactions:

$$\begin{cases} n \leftrightarrow p + e^- + \bar{\nu}_e \\ \nu_e + n \leftrightarrow p + e^- \\ e^+ + n \leftrightarrow p + \bar{\nu}_e \end{cases}$$

At $T\sim 1\,\text{MeV}$ protons and neutrons are non-relativistic, so

$$\frac{n_n}{n_p} = \mathrm{e}^{-(m_n - m_p)/T} = \mathrm{e}^{-(1.29\,\text{MeV})/T}$$

This decreases with T but

$$\Gamma(\nu_e + n \leftrightarrow p + e^-) \sim 2.1\left(\frac{T}{1\,\text{MeV}}\right) s^{-1}$$

which falls below H when $T\sim 0.8\,\text{MeV}$. The neutron abundance is frozen at

$$\frac{n_n}{n_p} \sim e^{-1.29/0.8} \sim 0.2$$

Neutrons can still decay but the lifetime is too long and instead they combine into helium via

$$\begin{aligned} &p + n \leftrightarrow d + \gamma \\ &d + d \leftrightarrow {}^3\mathtt{He} + n \\ &{}^3\mathtt{He} + d \leftrightarrow {}^4\mathtt{He} + p \end{aligned}$$

or

$$
\begin{aligned}
d+d &\leftrightarrow {}^3\mathtt{H}+p\\
{}^3\mathtt{H}+d &\leftrightarrow {}^4\mathtt{He}+n
\end{aligned}
$$

Since the photon density was so large photo-disintegration of deuterium was very efficient and hence the $d + d$ reactions went very slowly. Below 0.1 MeV photo-disintegration became inefficient and led to rapid $d+d$ fusion of helium. This consumed most of the neutrons so:

$$Y({}^4\mathtt{He}) = \frac{4n_{\mathtt{He}}}{n_{\mathrm{tot}}} \simeq \frac{4(n_n/2)}{n_n+n_p} = \frac{2\,n_n/n_p}{1+n_n/n_p} \simeq$$

(At the end of nucleosynthesis ($T \sim 0.01\,\mathrm{MeV}$), $n_n/n_p = 0.13$.)

$$\simeq 0.24$$

Some ${}^3\mathtt{He}$ and $\mathtt{D}$ also remain. Also ${}^7\mathtt{Li}$ is predicted via

$$
\begin{cases}
{}^4\mathtt{He}+{}^3\mathtt{H} \leftrightarrow {}^7\mathtt{Li}+\gamma\\
{}^4\mathtt{He}+{}^3\mathtt{H} \leftrightarrow {}^7\mathtt{Be}+\gamma\\
{}^7\mathtt{Be}+n \leftrightarrow {}^7\mathtt{Li}+p
\end{cases}
$$

All this leads to $\Omega_B h^2 \sim 0.02$.