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$$\rho \simeq \frac{\pi^2}{30}\, g_{\text{eff}}\, T^4 \quad \text{where } g_{\text{eff}} = \left(\sum_{\substack{i \\ \text{relativistic} \\ \text{bosons}}} g_i \left(\frac{T_i}{T}\right)^4 \right) + \frac{7}{8} \times \left(\sum_{\substack{i \\ \text{relativistic} \\ \text{fermions}}} g_i \left(\frac{T_i}{T}\right)^4 \right)$$

$$s = \frac{S}{V} = \frac{2\pi^2}{45}\, g^s_{\text{eff}}\, T^3 \quad \text{where } g^s_{\text{eff}} = \left(\sum_{\substack{i \\ \text{relativistic} \\ \text{bosons}}} g_i \left(\frac{T_i}{T}\right)^3 \right) + \frac{7}{8} \times \left(\sum_{\substack{i \\ \text{relativistic} \\ \text{fermions}}} g_i \left(\frac{T_i}{T}\right)^3 \right)$$

Decoupling of neutrinos

Weakly interacting particles like neutrinos decouple at a temperature T_{dec} below which the neutrino interactions are not fast enough to keep up with the Hubble expansion. The neutrinos interact via the weak interactions i.e. via exchange of $W^{\pm}$, Z. At temperatures much smaller than $80 - 90$ GeV (mass of $W^{\pm}$, Z) the propagators of $W^{\pm}$, Z go like $1/m_W^2$. The interaction rate is $\Gamma = \sigma\ |\boldsymbol{v}|\ n$, where σ is the cross section, $\boldsymbol{v}$ is the velocity and n is the number density. We study relativistic particles, where $|\boldsymbol{v}| \simeq c = 1$ and $n \propto T^3$ and $\sigma = \alpha^2\ s/m_W^4 \sim \alpha^2 E^2/m_W^4 \sim a^2 T^2/m_W^4$.

Thus $\Gamma_{\text{weak}} \sim \alpha^2 T^5/m_W^4$. From before $H \sim T^2/m_{\text{Pl}}$. Decoupling occurs when $\Gamma \simeq H$, i.e.

$$\frac{T_{\text{dec}}^2}{m_{\text{Pl}}} \simeq \frac{\alpha^2 T_{\text{dec}}^5}{m_W^4}, \quad \Rightarrow \quad T_{\text{dec}} \simeq \left(\frac{m_W^4}{\alpha^2\, m_{\text{Pl}}}\right)^{1/3} \sim 4\,\text{MeV}$$

After decoupling the neutrinos will move like free particles, only being affected by the general Hubble expansion. Just like photons they will be redshifted by a/a_{dec} i.e.

$$T_\nu = T_{\text{dec}}\left(\frac{a_{\text{dec}}}{a}\right)$$

Recall that for particles in thermal equilibrium entropy is conserved: $s = g_{\text{eff}}\ (a\ T)^3 = \text{const} \ \Rightarrow T \sim (g^s_{\text{eff}})^{-1/3}\, a^{-1}$.

Hence, provided g^s_{eff} does not change the neutrino distribution will still look like it is in thermal equilibrium.

However, g^s_{eff} will change when the electrons (and positrons) become non-relativistic and annihilate via $e^+ + e^- \to \gamma\,\gamma$. This will happen around $1\text{ MeV} \approx 2\, m_e$, since below this energy $\gamma\gamma \to e^+ + e^-$ is no longer kinematically possible.

At temperatures a bit above 1 MeV the relativistic species in thermal equilibrium are γ, e^+, e^- (the neutrinos have already decoupled) leading to

$$(g^s_{\text{eff}})_{\text{before}} = \underset{\gamma}{2} \;+\; \underset{e^{\pm}}{2 \times 2 \times \frac{7}{8}} \;=\; \frac{11}{2}$$

Below 1 MeV only the γ's are still in thermal equilibrium, giving $(g^s_{\text{eff}})_{\text{after}} = 2$. Since the entropy for equilibrium particles is conserved:

$$(g^s_{\text{eff}})_{\text{before}}\, (a\,T)^3_{T>1\text{MeV}} = (g^s_{\text{eff}})_{\text{after}}\, (a\,T)^3_{T<1\,\text{MeV}}$$

$$(a\,T)_{T<1\,\text{MeV}} = \left(\frac{11}{4}\right)^{1/3} (a\,T)_{T>1\text{MeV}} \simeq 1.4\, (a\,T)_{T>1\text{MeV}}$$

The entropy transfer from the decoupling e^+e^- to the photons is called reheating (although the temperature for the photons only decreases less rapidly, it does not rise). The neutrinos do not benefit from the reheating, so

$$T_\nu = \left(\frac{4}{11}\right)^{1/3} T_\gamma$$

Since there is a background photon radiation with $T_\gamma^{\mathrm{today}} \simeq 2.73$ K there should also be a neutrino background with $T_\nu^{\mathrm{today}} = (4/11)^{1/3} \times 2.73\,\mathrm{K} \approx 1.95\,\mathrm{K}$. Has not been observed: a challenge!

The total radiation energy density today can be obtained be obtained [*sic*] by using

$$g_{\mathrm{eff}}^{\mathrm{today}} = 2 + \frac{7}{8} \times 2 \times 3 \times \left(\frac{4}{11}\right)^{4/3} \simeq 3.36$$

so

$$\rho_{\mathrm{R}}^{\mathrm{now}} = \frac{\pi^2}{30}\, g_{\mathrm{eff}}^{\mathrm{today}} \left(T_\gamma^{\mathrm{today}}\right)^2 \sim 8.1 \times 10^{-34}\,\mathrm{g/cm}^3$$

which corresponds to $\Omega_{\mathrm{CMBR}} h^2 \sim 4.3 \times 10^{-5}$.

What about gravitons?

$$\sigma_{\mathrm{grav}} \sim \frac{T^2}{m_{\mathrm{Pl}}^4} \quad \Rightarrow \quad \frac{\Gamma_{\mathrm{grav}}}{H} \sim \frac{T^3}{m_{\mathrm{Pl}}^3}$$

so decoupling temperature is enormous: $T \sim m_{\mathrm{Pl}} \sim 10^{19}$ GeV. At this Planck scale there were probably more relativistic degrees of freedom than the 106.75 of the standard model.

$$T_{\mathrm{grav}} = \left(\frac{(g_{\mathrm{eff}}^s)^{\mathrm{now}}}{(g_{\mathrm{eff}}^s)^{\mathrm{Planck}}}\right)^{1/3} T_\gamma^{\mathrm{now}} < \left(\frac{3.4}{106.8}\right)^{1/3} 2.73 \approx 0.9\,\mathrm{K}$$

Very, very hard to detect.

Matter–antimatter asymmetry (Baryogenesis)

After the annihilation of electrons and positrons (i.e. $T \ll 1\,\mathrm{MeV}$) there was a left over excess of electrons enough to balance the electric charge of the protons. The origin of the asymmetry between matter and antimatter is unknown. In particle physics there exists mechanisms which may create such an asymmetry, but the details of how this would work are not understood.

Sakharov has shown that in order to obtain a matter–antimatter asymmetry in the universe (even from a symmetric initial state) three conditions are necessary:

- Baryon number violation.

This is clear since today the number of baryons (neutrons and protons) is much larger than the number of anti-baryons. Assuming that the number of baryons and anti-baryons was (almost) equal initially baryons number cannot be conserved.

- Deviation from thermal equilibrium.

The mass of a particle is the same as that of its anti-particle (this follows from CPT which is believed to be an exact symmetry in nature). The number densities of a baryon and an anti-baryon in thermal equilibrium are the same (since they depend on the same mass) but with opposite chemical potentials. However, since baryon number is not conserved $\mu = 0$ (extremizes entropy) so in thermal equilibrium $n_B = n_{\bar{B}}$ would hold. Thus we need departure from thermal equilibrium.

- C and CP violation.

Both C violation and CP violation are needed for the generation of a baryon–antibaryon asymmetry. The baryon number is odd under both C and CP.