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Lecturer: Niclas Wyllard

Phase transitions

Phase: Uniform state of a system where the physical properties are the same everywhere (also for part of a system). Two types of phase transitions:

1. Non-continuous transitions. (Sometimes you see the terminology "first order".) Are characterised by an (intensive) thermodynamical variable that is discontinuous. Transitions where heat is emitted or absorbed (latent heat) are non-continuous.

Example: Boiling water (Density as a function of the pressure and temperature is discontinuous.)

2. Continuous transitions (second order). No latent heat. (Critical exponents.)

Example:

— Transition to a ferromagnetic phase in a metal.

— Transition to superconducting phase.

Motivation

Since the universe started out in a very hot state, there could have been many phase transitions as the universe evolved (cooled).

For any phase transition one can (for the other parameters fixed) identify a critical temperature, T_c. For a second order phase transition the correlation length ξ (size of the phase) scales as $\xi = \xi_0(T - T_c)^{-\nu}$ where $\nu > 0$ is an example of a critical exponent.

An order parameter is a quantity from which we can infer which phase we are in (e.g. the magnetisation for the ferromagnetic transition).

Landau description of phase transitions

In this approach one writes down an effective field theory which respects the symmetries of the problem. The field represents the order parameter in the following sense: the minima (vacua) of the potential correspond to the different phases and $\langle\phi\rangle = \phi_{\min}$ gives the order parameter in the different phases.

Examples

$$\mathcal{L}(x) = \partial_\mu\phi\,\partial^\mu\phi + \underbrace{\mu^2\,\phi^2 - \frac{\lambda}{2}\,\phi^4}_{V(\phi)}$$

The vacua (minima, $V'(\phi) = 0$) are $\phi^2 = \mu^2/\lambda$, i.e. $\phi_\pm = \pm\sqrt{\mu^2/\lambda}$. Assume now that one part of the "universe" went into a state corresponding to ϕ_+ and one went into a state corresponding to ϕ_-.

The region where the two phases meet has to involve a smooth transition over a region of size δ, say. (A sharp transition would involve infinite amounts of energy, since $\partial_\mu\phi$ diverges.) This region of size δ is called a domain wall. Assume it is located in the $x = 0$ plane. The equation of motion across the plane is then:

$$\frac{\partial^2\phi}{\partial x^2} - \lambda\left(\phi^2 - \frac{\mu^2}{\lambda}\right) = 0$$

with boundary conditions

$$\begin{cases} \phi(x \to \infty) = \phi_- \\ \phi(x \to -\infty) = \phi_+ \end{cases}$$

This equation has the so called kink solution which characterises the domain wall:

$$\phi = \sqrt{\frac{\mu^2}{\lambda}} \tanh\left(\frac{x}{\delta}\right)$$

where the width parameter

$$\delta = \sqrt{\frac{2}{\mu^2}}.$$

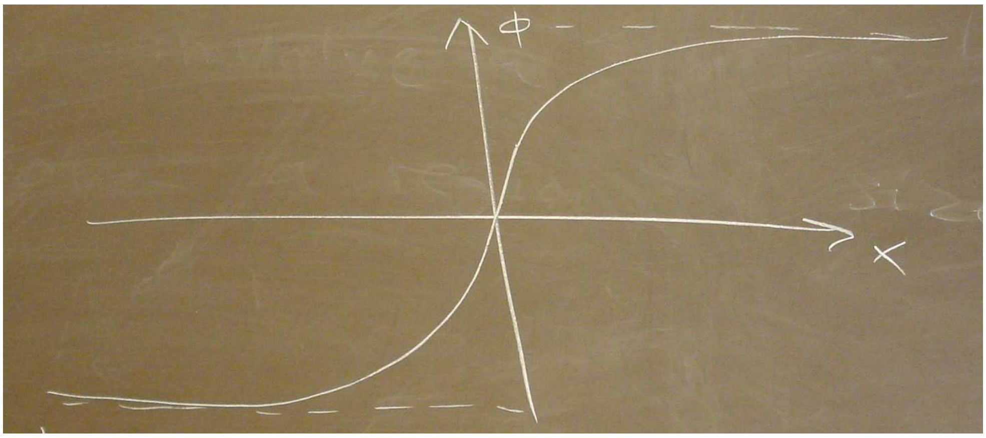

Figure 1. The domain wall.

Above we considered an effective theory involving a single scalar field and found domain wall solutions: solutions which involve planes (two-dimensional surfaces) separating different phases. For effective theories involving more fields, more exotic solutions are possible.

For two scalar fields (or one complex field) the minima are

$$|\phi|^2 = \phi_1^2 + \phi_2^2 = \frac{\mu^2}{\lambda}$$

(c.f. Higgs model)

$$\langle \phi \rangle = \phi_{\min} = \sqrt{\frac{\mu^2}{\lambda}}\, \mathrm{e}^{\mathrm{i}\theta(x)}$$

The field $\phi(x)$ must be single valued so whenever we go around a circle in x-space, $\theta(x)$ must change by $\Delta\theta = 2\pi n$, $n \in \mathbb{Z}$. Assume that we have a path with $\Delta\theta = 2\pi$. When we shrink this path to a point we cannot reach $\Delta\theta = 0$ unless we cross a point with $\langle \phi \rangle = 0$. This is part of a tube. In most cases the length is much greater than the transverse dimension, so the tubes look like strings.

Number of fields N:

$$\begin{array}{lll} N = 1 & \rightarrow & \text{membranes (domain walls)} \\ N = 2 & \rightarrow & \text{strings} \\ N = 3 & \rightarrow & \text{monopoles (particles)} \\ N = 4, 5, \ldots & \rightarrow & \text{textures: knots, etc.} \end{array}$$

These textures are *not* stable. In our universe monopoles and domain walls are ruled out, but (cosmic) string defects and textures may occur.