## 2009–02–04

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Last time: the particles in the standard model.

Spin 1: $\gamma, W^{\pm}, Z$, gluons. Spin $\frac{1}{2}$: $e, \nu_e, \mu, \nu_\mu, \tau, \nu_\tau, u, d, s, c, t, b+$ antiparticles. Spin 0: Higgs?

**Feynman diagrams**

So far we have not talked much about interactions and quantum corrections. There is a very useful method for computing quantum corrections, using so called Feynman diagrams. The rules for constructing these diagrams can be read off directly from the Lagrangian.

Let's start with something simple. Consider the "partition function":

$$Z = \int_{-\infty}^{\infty} \exp\left( -\frac{m^2}{2}\varphi^2 + \lambda\varphi^4 \right) \mathrm{d}\varphi$$

This is divergent. But we don't care about that. We do a formal power series:

$$Z = \sum_{n=0}^{\infty} \frac{1}{n!}\lambda^n \int_{-\infty}^{\infty} \varphi^{4n} \exp\left( -\frac{m^2}{2}\varphi^2 \right) \mathrm{d}\varphi.$$

These integrals are convergent.

$$Z = \sum_{n=0}^{\infty} \frac{1}{n!}\lambda^n \left( \frac{\mathrm{d}}{\mathrm{d}J} \right)^{4n} \int_{-\infty}^{\infty} \exp\left( -\frac{m^2}{2}\varphi^2 + \varphi J \right) \mathrm{d}\varphi \bigg|_{J=0}.$$

$$\exp\left( -\frac{m^2}{2}\varphi^2 + \varphi J \right) = \exp\left( -\frac{m^2}{2}\left( \varphi - \frac{J}{m^2} \right)^2 + \frac{J^2}{2m} \right)$$

$$Z = \left( \int_{-\infty}^{\infty} \mathrm{d}\varphi \, \exp\left( -\frac{m^2}{2}\varphi^2 \right) \right) \sum_{n=0}^{\infty} \frac{1}{n!}\lambda^n \left( \frac{\mathrm{d}}{\mathrm{d}J} \right)^{4n} \exp\left( \frac{J^2}{2m^2} \right) \bigg|_{J=0}$$

$\frac{\mathrm{d}}{\mathrm{d}J}$ need to come “in pairs”, since otherwise factors of $J$ will appear that are killed by $\Big|_{J=0}$.

Represent the expansion graphically:

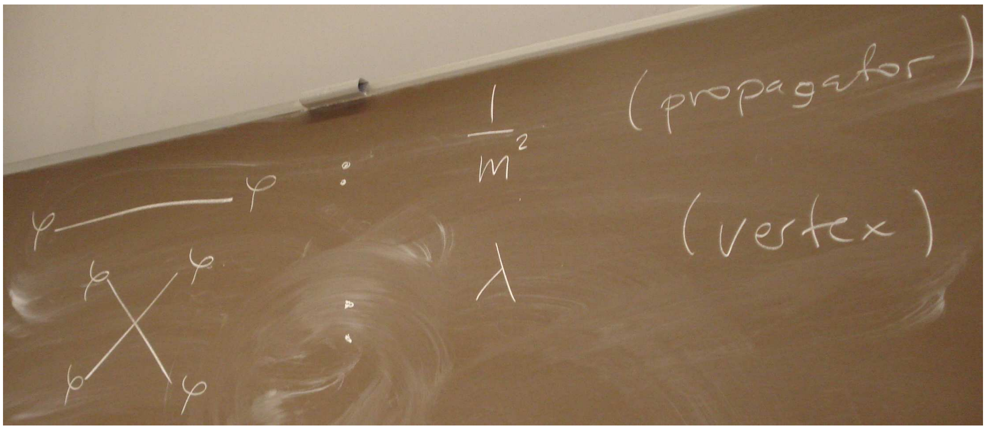


**Figure 1.** Propagators and vertices.

Note that these can be related to the “Lagrangian” $L = -\frac{m^2}{2}\,\varphi^2 + \lambda\,\varphi^4$: The propagator is the inverse of the “operator” $x$ in $-\frac{1}{2}\varphi\,x\,\varphi$. The vertex follows from the $\lambda\varphi^4$ term. The 4 in the $\lambda\varphi^4$ tells us that there are four lines going out from/into the vertex.

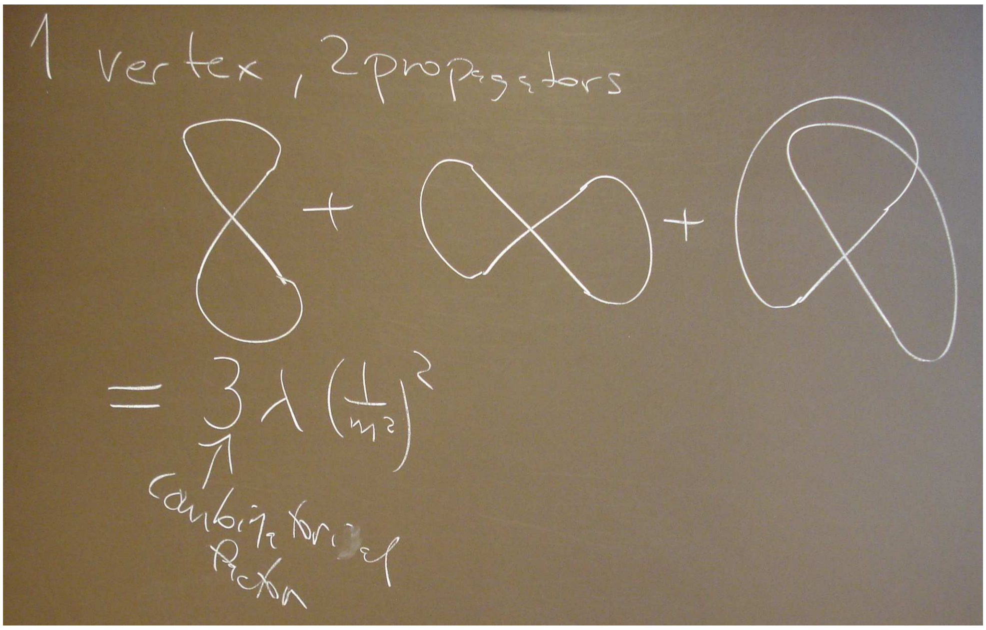


**Figure 2.** One vertex, two propagators.

This is faster than doing the derivatives, but gives the same thing. The next thing would be two vertices — then we would need four propagators. (That's 105 possibilities. $7 \times 5 \times 3 = 105$.) You are not supposed to have any free ends sticking out. We get

$$\frac{105}{2!} \frac{\lambda^2}{(m^2)^4}$$

The 2! is due to the equivalence of the two vertices.

Some diagrams are disconnected, e.g. 8∞⟨fig⟩, others are connected, e.g. ⟨fig⟩.

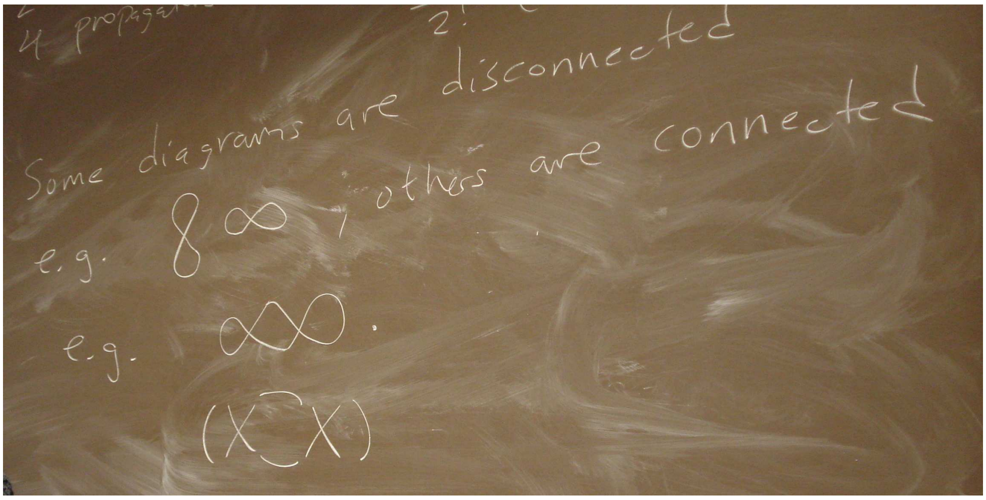


**Figure 3.**

Consider

$$W = \ln Z = \text{const} - \frac{1}{2} \ln m^2 + \frac{3\lambda}{m^4} + \frac{(105 - 9)}{2} \frac{\lambda^2}{m^8} + \cdots$$

$$\int_{-\infty}^{\infty} \mathrm{d}\varphi \exp\left( - \frac{m^2}{2} \varphi^2 \right) = \frac{\sqrt{2\pi}}{\sqrt{m^2}} = \exp\left( \text{const} - \frac{1}{2} \ln m^2 \right)$$

$W$ is an expansion in *connected* diagrams. Check: for two vertices there are $3 \times 3$ disconnected diagrams. $W$ is an expansion in the number of loops:

| | |
|---|---|
| $-\frac{1}{2} \ln m^2$: | 1-loop term |
| $3\frac{\lambda}{m^4}$: | 2-loop term |
| $\frac{96}{2} \frac{\lambda^2}{m^8}$: | 3-loop term |

So far we have only calculated "vacuum diagrams", i.e. diagrams with no external legs. Diagrams with external legs are also of interest: they correspond to "scattering" diagrams.

**Figure 4.** Two things come in, something happens in the middle, two things go out. These "things" will eventually be found to be particles.

**Figure 5.** Expansion of "something happens in the middle".

Such diagrams arise from

$$\langle \varphi^n \rangle = \int_{-\infty}^{\infty} \mathrm{d}\varphi \, \varphi^n \exp\!\left( -\frac{m^2}{2}\varphi^2 + \lambda\varphi \right).$$

(We are not quite following the book. But you will need to read the book as well, in order to solve the hand-in problems.)

In quantum field theory one has a similar structure. For a spin 0 scalar field one has

$$\varphi \rightarrow \phi(x)$$

$$-\frac{m^2}{2}\varphi^2 + \lambda\varphi^4 \rightarrow \int \mathrm{d}^4x \left( \frac{1}{2}\,\partial_\mu\phi\,\partial^\mu\phi - \frac{m^2}{2}\phi^2 + \lambda\phi^4 \right)$$

$$\int \mathrm{d}\varphi \rightarrow \int \mathrm{D}\phi(x): \quad \text{this is a path integral.}$$

Note

$$\int \mathrm{d}^4x\,\partial_\mu\phi\,\partial^\mu\phi = -\int \mathrm{d}^4x\,\phi\,\partial_\mu\partial^\mu\phi.$$

The propagator is the inverse of $x$ in $-\frac{1}{2}\phi x \phi$, i.e. the inverse of $\partial_\mu\partial^\mu + m^2$. In momentum space (Fourier transform), $\partial_\mu$ is just $p_\mu$ (ignoring factors of i), so the inverse is $1/(p^2 + m^2)$, where $p^2 \equiv p_\mu p^\mu$. The vertex is $\lambda$ as before. There are also some $\delta$-functions which impose momentum conservation. For low energies ($p_\mu p^\mu$ small compared to $m^2$) the propagator is simply $1/m^2$.

The propagator for the vector (spin 1) fields is a bit complicated because of the vector index on $A_\mu$, $W_\mu^\pm$, $Z_\mu$, ... However, for the massive particles ($W^\pm$, $Z$) at low energies the propagator is simply $\eta_{\mu\nu}/m^2$:

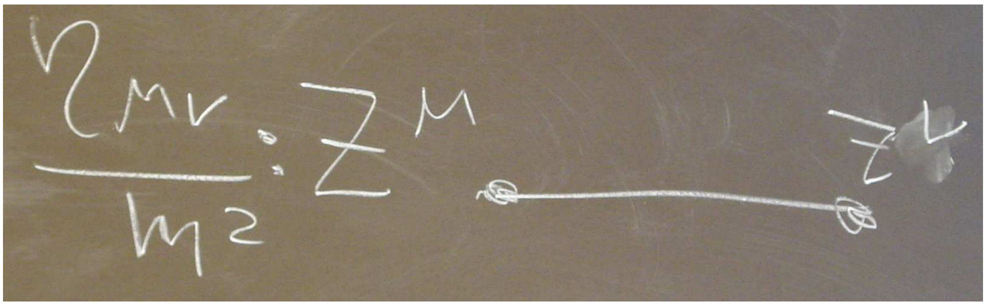


**Figure 6.** The propagator $\eta_{\mu\nu}/m^2$.

The propagator for the fermions (spin $\frac{1}{2}$) is also a bit complicated (it involves $\gamma^\mu$ etc).

Examples of interactions (vertices) in the standard model:

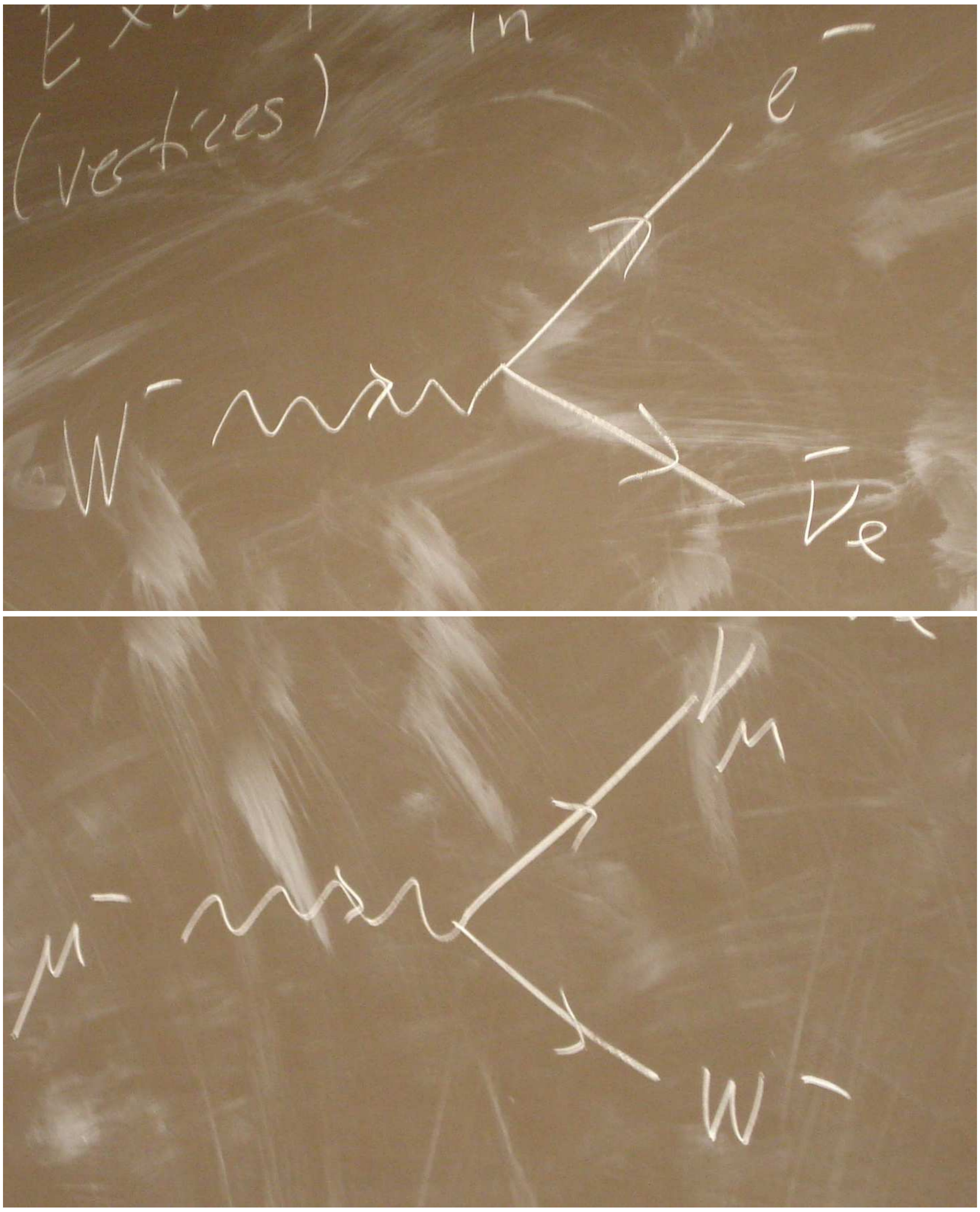


**Figure 7.** $W^- \to e^- + \bar{\nu}_e$ and $\mu^- \to \nu_\mu + W^-$.

Combining leads to muon decay:

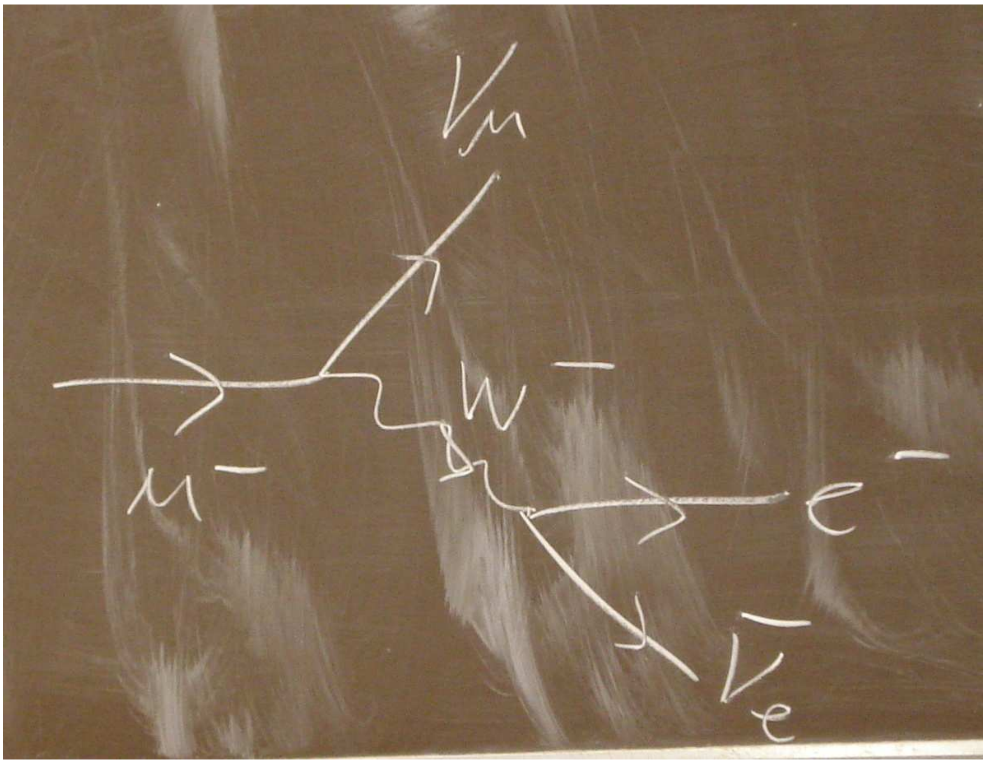


**Figure 8.** Muon decay: $\mu^- \to e^- + \bar{\nu}_e + \nu_\mu$

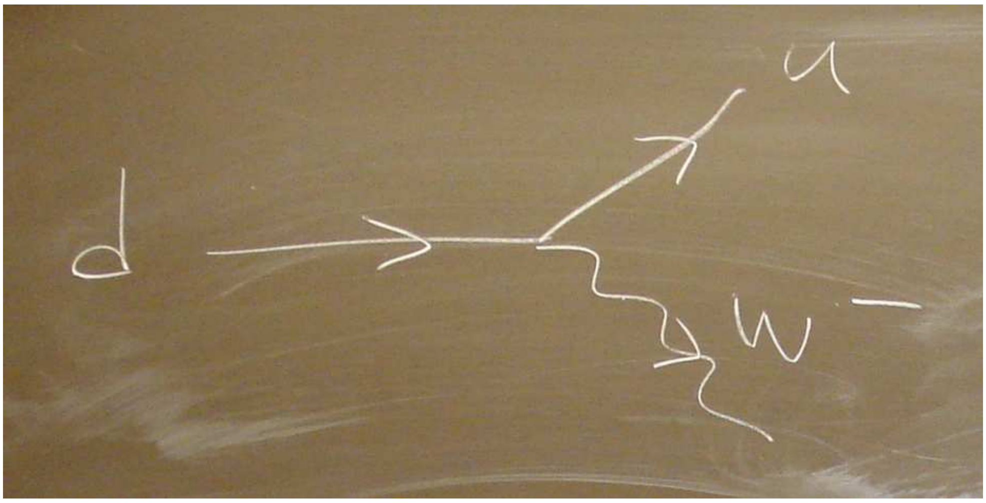


**Figure 9.** Down-quark decay. $d \to u + W^-$. (Such decays lead to $\beta$ decay: $n \to p + e^- + \bar{\nu}_e$.)


**Figure 10.** $\beta$-decay. $n \to p + e^- + \bar{\nu}_e$. (Here the quarks look like free particles, which they are not, so it is a bit simplified.)

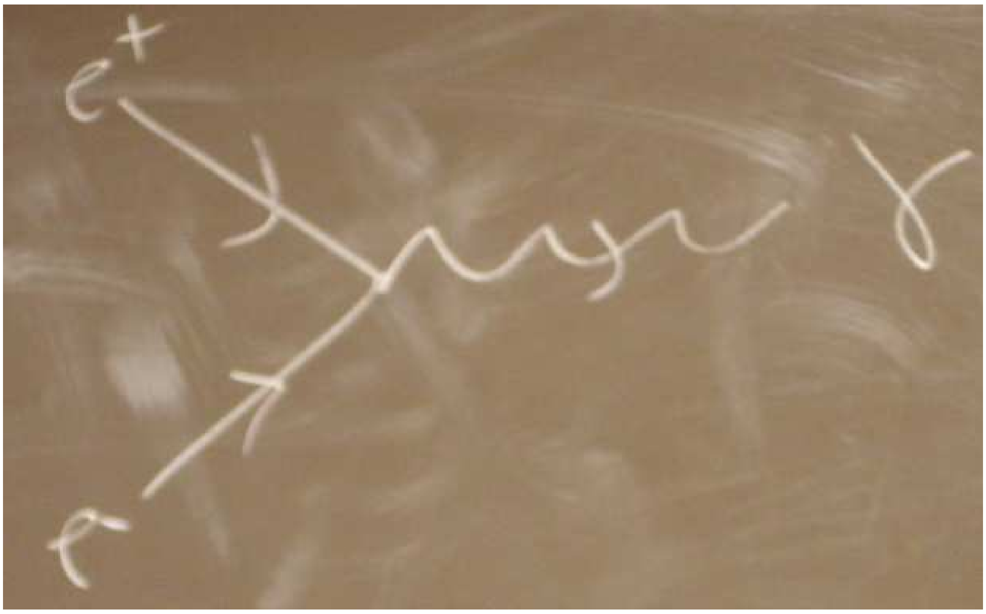


**Figure 11.** Electron-positron annihilation.

### Beyond the standard model

Despite all its successes (the best tested theory we have ever constructed), the standard model is incomplete. It does not include gravity. There should be some kind of quantum gravity. That's one motivation, but it is not the only one.

One extension which has some appealing features is supersymmetry. This model predicts a sypersymmetric partner to all particles in the standard model. So far, *none* of these have been observed in experiments.