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Lecturer: Niclas Wyllard

You find the course home page at `http://fy.chalmers.se/~wyllard/astro.html`. If you need to get in touch with me, write to `wyllard@chalmers.se`. That's the safest way to get in touch, I always check my email.

We plan when to have lectures. Ordinarily: Monday 15–17 and Wednesday 10–12. Extra lectures: Tuesday 15–17.

The courses "gravitation and cosmology" and "quantum field theory" are recommended, but not mandatory, for this course. This course is very broad, covering small pieces from different areas.

There will be four sets of homework problems. That is the main part of the exam. There will also be an oral exam at the end. The first batch of problems will be up in a couple of days, with a deadline of two weeks or something.

The chapters covered are (roughly) 1, 4, 6–11, 14. Parts of 2, 3 and appendices.

Goal & scope of the course: The goal is to give an overview of modern cosmology, highlighting the necessary role of fundamental theories of physics, such as the standard model of particle physics, in the description of of [*sic*] the early universe.

Units:

The first important set of units is the so called natural units. These are units where you set $c = \hbar = k_{\mathrm{B}} = 1$, where c is the speed of light, $\hbar$ is Planck's constant over 2π, and k_{B} is Boltzmann's constant. (Common in particle physics.)

One dimension remains:

$$[\text{energy}] = [\text{mass}] = [\text{temperature}] = [\text{time}]^{-1} = [\text{length}]^{-1}$$

Why: Energy: $m\,c^2, k_{\mathrm{B}}\,T, \hbar\omega$.

In particular in particle physics there is a common energy unit, which is the electron volt (eV).

$$1\,\mathrm{eV} \approx 0.16 \times 10^{-18}\,\mathrm{J}$$

(And, naturally, also with prefixes: keV, MeV, GeV, ...)

Dimensionless (Planck) units: $c = \hbar = k_{\mathrm{B}} = G = 1$, where G is Newton's constant.

From these units you can form...

- The Planck length:

$$l_{\mathrm{Pl}} = \left(\frac{\hbar G}{c^3}\right)^{1/2} \approx 1.6 \times 10^{-33}\,\mathrm{cm}$$

- The Planck mass:

$$m_{\mathrm{Pl}} = \left(\frac{\hbar c}{G}\right)^{1/2} \approx 2.2 \times 10^{-5}\ \mathrm{g}$$

This is a rather large mass, if you compare it with other typical masses.

- The Planck time:

$$t_{\mathrm{Pl}} = \left(\frac{\hbar G}{c^5}\right)^{1/2} \approx 5.4 \times 10^{-44}\ \mathrm{s}$$

A rather short time.

• The Planck temperature:

$$T_{\mathrm{Pl}} = \frac{m_{\mathrm{Pl}}\, c^2}{k_{\mathrm{B}}} \approx 1.4 \times 10^{32}\,\mathrm{K} = 1.2 \times 10^{9}\,\mathrm{GeV}$$

which is rather hot.

At the Planck length/time/temperature neither quantum mechanics, nor Einstein's theory of gravitation can be neglected. What we need is a theory of quantum gravity. The bad news, is that nobody knows such a theory.

Other useful units:

1 pc (parsec) = 3.26 light years = $3.09 \times 10^{18}\,\mathrm{cm}$.

The symbol $\odot$ is used to indicate solar units, so that for instance $M_{\odot} =$ the mass of the sun $\approx 2 \times 10^{33}\,\mathrm{g}$.

This course is a mainly theoretical course, but close to the experimental findings. This is an active field of study.

Observational foundations of cosmology

• Our universe is isotropic and homogeneous on scales larger than 100 Mpc (that's very large) up to at least 3000 Mpc. On smaller scales, the universe is inhomogeneous (clusters of galaxies, galaxies, stars, ...)

The uniformity of the universe is very important since it means that the universe we observe will "look the same" as elsewhere in the universe. Our observations thus give us clues about the universe as a whole.

• The universe is expanding.

• There is a background radiation of photons corresponding to a black body radiation with $T \approx 2.7\,\mathrm{K}$. This radiation is uniform to a very high degree, both in observational angle and in temperature. There are slight variations in the background radiation, that are actually very important — we'll come back to that later. This uniformity is a good test of the isotropy and homogeneity of the universe.

• There is roughly one neutron/proton ("baryons") per 10^9 photons. There are many more photons than you have neutrons and protons. There is no significant amount of antimatter.

• The matter in our universe is composed of $\sim 75\%$ hydrogen and $\sim 25\%$ helium, and trace amounts of other (heavier) elements.

Hot Big Bang model

The observational data are consistent with the so called hot Big Bang model, which states that the universe has a finite age (roughly 13 to 14 billion years). The universe was in the beginning very small / compressed, and consequently very hot. Since then, the universe has expanded and cooled (and continues to do so today). In the beginning the universe was composed of elementary particles (photons, electrons, quarks, ...)

As the universe cooled, bound states started to form: protons and neutrons, nuclei, atoms, molecules, stars, ...

The age of the universe is consistent with the ages of the oldest stars in our galaxy. (Our solar system is about 4.5 billion years.)

The expansion and Hubble's law

In an expanding (homogeneous and isotropic) universe the relative velocity of an observer B with respect to an observer A obey Hubble's law.

$$\boldsymbol{v}_{\mathrm{BA}} = H(t)\, \boldsymbol{r}_{\mathrm{BA}} \quad \text{(Newtonian)}$$

$H_0 = H(t_0)$, where t_0 refers to the present time. H is Hubble's "constant". According to observations:

$$H_0 = h \cdot 100\,\mathrm{km\,s^{-1}\,Mpc^{-1}}$$

where h (introduced for historical reasons) is about $0.719^{+0.026}_{-0.027}$.

Dark matter

Example. In an expanding universe with $v = H_0\, r$, show that a massive particle outside a spherical piece of the universe with radius r, will escape the gravitational attraction if the density $\rho_M \leqslant \rho_\mathrm{c} = 3\, H_0^2/8\pi G$.

According to Newtonian mechanics the particle will escape if

$$\frac{m\, v^2}{2} - \frac{G\, M m}{r} = 0.$$

With $v = H_0\, r$ and $M = 4\pi\rho_\mathrm{c}\, r^3/3$, we find $\rho_\mathrm{c} = 3\, H_0^2/8\pi G$. This is the critical density.

Definition: $\Omega = \rho/\rho_\mathrm{c}$.

Now, going back to dark matter.

Dark mater

For baryons (neutrons and protons) observations give $\Omega_B\, h^2 \approx 0.02273 \pm 0.00062$. On the other hand the total matter density is $\Omega_m\ h^2 \approx 0.1326 \pm 0.0063$, which leads to the dark matter problem: What makes up the extra matter responsible for the discrepancy between these two?

The cosmological constant

In addition to the energy density associated with ordinary matter, there may also be an energy density associated with a so called cosmological constant, usually denoted by Λ. While the presence of matter slows the expansion, a cosmological constant with $\Omega_\Lambda = \Lambda/3\, H_0^2 > 0$ increases the expansion rate. Observations indicate $\Omega_\Lambda > 0$. This is either the cosmological constant, or some other sort of dark energy.