2008–09–23

$$A(z) = 2\pi \int_{z_1}^{z_2} \rho(z)\, \sqrt{1 + \rho'(z)}\, \mathrm{d}z$$

$\rho(z)$. $z \to t$, $\rho \to q$. We may treat the integrand as the Lagrangian L.

$$\frac{\partial L}{\partial \rho_0} - \frac{\mathrm{d}}{\mathrm{d}z}\left(\frac{\partial L}{\partial \rho'}\right) = 0$$

$$J = \rho' \frac{\partial L}{\partial \rho'} - L = \text{constant} = \rho'\, \rho \frac{2\rho'}{2\sqrt{1+\rho'^2}} - \rho\sqrt{1+\rho'^2} = \frac{\rho\, \rho'^2}{\sqrt{1+\rho'^2}} - \rho\,\sqrt{1+\rho'^2} =$$

$$= -\frac{\rho}{\sqrt{1+\rho'^2}}$$

$$J = \frac{\rho}{\sqrt{1+\rho'^2}} = \text{constant}$$

$$J^2 = \frac{\rho^2}{1+\rho'^2}$$

$$J^2(1+\rho'^2) = \rho^2$$

$$\rho'^2 = \frac{\rho^2}{J^2} - 1$$

$$\rho' = \pm\sqrt{\frac{\rho^2}{J^2} - 1}$$

$$\frac{\mathrm{d}\rho}{\mathrm{d}z} = \sqrt{\frac{\rho^2}{J^2} - 1}$$

$$\frac{\mathrm{d}\rho}{\sqrt{\frac{\rho^2}{J^2} - 1}} = \mathrm{d}z$$

[Andro is constantly standing in front of what he has written, so that I can't see it as he writes it. And now he erased some equations that I haven't written down, because I have to wait for him to move before writing anything.]

$$J\ln\frac{1}{J}\left(\rho+\sqrt{\rho^2-J^2}\right)=z+C$$

$$\frac{1}{J}\left(\rho+\sqrt{\rho^2-J^2}\right)=\mathrm{e}^{(z+C)/J}$$

$$J\,\mathrm{e}^{(z+C)/J}-\rho=\sqrt{\rho^2-J^2}$$

$$J^2\,\mathrm{e}^{2^{\frac{z+C}{J}}}+\rho^2-2\,J\rho\,\mathrm{e}^{\frac{z+C}{J}}=\rho^2-J^2$$

$$J\left(1+\mathrm{e}^{2^{\frac{z+C}{J}}}\right)=2\,\rho\,\mathrm{e}^{\frac{z+C}{J}}$$

$$\rho=\frac{J}{2}\left(\mathrm{e}^{-\frac{z+C}{J}}+\mathrm{e}^{\frac{z+C}{J}}\right)=J\cosh\left(\frac{z+C}{J}\right)$$

$$\rho=J\cosh\left(\frac{-z+C'}{J}\right)$$

$$\rho=J\cosh\left(\frac{z+C}{J}\right),\quad \rho'>0$$

$$\rho=J\cosh\left(\frac{-z+C'}{J}\right),\quad \rho'<0$$

$$\rho'\ldots$$

$\rho'=0$ where $z=z_0=-C$, $C'=-C$.

$$\rho(z)=J\cosh\left(\frac{z+\rho}{J}\right)$$

$\rho(z_1)=\rho(z_2)=a$.

$$\begin{cases}\frac{a}{J}=\cosh\left(\frac{z_1+C}{J}\right)\\ \frac{a}{J}=\cosh\left(\frac{z_2+C}{J}\right)\end{cases}$$

[OK, Andro's soporific voice wins. I couldn't possibly concentrate on what he is saying. Now, I'm off to check my mail.]